Fuzzy Bags

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Abstract: *This study introduces a revised definition for fuzzy bags. It is based on the definition of bags given by Delgado et al. 2009 in which each bag has two parts, function and summary information. Furthermore, the concept of α-cuts and related theorems is given. By some examples, the new concepts are illustrated.*

Keywords: *α-cut of fuzzy bags; bags; fuzzy bags*

1 Introduction

The initial notion of bags was introduced by Yager [1] as an algebraic set-like structure where an element can appear more than once. So far, several works have been done using this new concept. Also, bags have been employed in practice, for instance; in flexible querying, representation of relational information, decision problem analysis, criminal career analysis, and even in fields such as biology.

However, due to some existing drawbacks in the first definition of bags [1], the necessity of a revision of this notion reveals. The proposed definition by Delgado et al. [2] has improved these drawbacks. By some examples, they showed that Yager's definition for bags has some deficiencies and it was not well suited for representing and reasoning with real-world information. Then, they proposed new definitions for bags and fuzzy bags.

As it is shown in [3], the lattice of all fuzzy bags defined by Delgado et al. [2] is a complete Boolean algebra which is not compatible with the nature of fuzziness. Improving this incompatibility, in this paper, we introduce a revised definition for fuzzy bags based on the proposed definition of bags in [2].

2 Preliminaries

In this section, some basic concepts which are needed in the sequel are given. For more details, see [2].

Definition 1. [2] Let P and O be two universes (sets) called "properties" and "objects", respectively. A (crisp) bag $\mathcal{B}^f$ is a pair (f, B^f), where $f : P \to \mathcal{P}(O)$ is a function and B^f is the following subset of $P \times \mathcal{N}$

$$B^f = \{(p, card(f(p)))|p \in P\}.$$

Here, $\mathcal{N}$ is the set of natural numbers, $\mathcal{P}(O)$ is the power set of O, $card(X)$ is the cardinality of set X.

We will use the convention here that $card(\emptyset) = 0$.

In this characterization, a bag $\mathcal{B}^f$ consists of two parts. The first one is the function f that can be seen as an information source about the relation between objects and properties. The second part B^f is a summary of the information in f obtained by means of the count operation $card(.)$. This summary corresponds to the classical view of bags in the sense of [1].

Notation 1. We set $\mathbf{B}(P, O)$ as the set of all bags $\mathcal{B}^f = (f, B^f)$ defined in Definition 1.

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Definition 2. Define $\mathcal{B}^0 = (0, B^0)$ and $\mathcal{B}^1 = (1, B^1)$ where, $0(p) = \emptyset$, $1(p) = O$ for all $p \in P$, $B^0 = \{(p, 0), p \in P\}$ and $B^1 = \{(p, card(O)), p \in P\}$. Clearly, $\mathcal{B}^0, \mathcal{B}^1 \in \mathbf{B}(P, O)$.

Example 1. [2] Let $O = \{$John, Ana, Bill, Tom, Sue, Stan, Ben$\}$ and $P = \{17, 21, 27, 35\}$ be the set of objects and the set of properties, respectively. Let $f : P \to \mathcal{P}(O)$ be the function in Table 1 with $f(p) \subseteq O$ for all $p \in P$.

Table 1: Function: age-people.

p	17	21	27	35
$f(p)$	$\{\text{Bill}, \text{Sue}\}$	$\{\text{John}, \text{Tom}, \text{Stan}\}$	$\emptyset$	$\{\text{Ben}\}$

So, we can define bag $\mathcal{B}^f = (f, B^f)$ where, $B^f = \{(17, 2), (21, 3), (27, 0), (35, 1)\}$.

In the next section, we introduce the concept of fuzzy bags and give some results about them.

3 Fuzzy Bags

In what follows, O is the set of all objects, and $\mathcal{F}(O) = \{A | A : O \to [0, 1]\}$ is the set of all fuzzy subsets of O. Also, $i \in I_n = \{1, 2, \ldots, n\}$, where $n \in \mathcal{N}$ and $\mathcal{N}$ is the set of natural numbers.

Definition 3. A fuzzy bag $\tilde{\mathcal{B}}^{\tilde{f}}$ is a pair $(\tilde{f}, B^{\tilde{f}})$, where $\tilde{f} : P \to \mathcal{F}(O)$ is a function and B^f is the following subset of $P \times [0, 1] \times \mathcal{N}$

$$B^{\tilde{f}} = \{(p, \delta, card(O^p_\delta)) | p \in P, \delta \in [0, 1]\}.$$

Where, $O^p_\delta = \{o \in O | \tilde{f}(p)(o) = \delta\}$.

Clearly, a crisp bag is a particular case of fuzzy bag where, for all $p \in P$, $\tilde{f}(p)$ is a crisp subset of O. Here, the concept of fuzzy bag is illustrated by an example.

Example 2. Let $O = \{\text{Ben}, \text{Sue}, \text{Tom}, \text{John}, \text{Stan}, \text{Bill}, \text{Kim}, \text{Ana}, \text{Sara}\}$ and $P = \{\text{young}, \text{middle age}, \text{old}\}$ is the set of some linguistic descriptions of age. Let the degrees of membership of all $o \in O$ in the set of each property $p \in P$ are given as in Table 2.

Table 2: The degrees of memberships for Example 2

p \ o	Ben	Sue	Tom	John	Stan	Bill	Kim	Ana	Sara
young	0.7	0.2	0.4	0.0	0.7	0.4	0.2	0.7	0.1
middle age	0.3	0.8	0.7	0.3	0.3	0.7	0.8	0.3	0.5
old	0.1	0.2	0.1	0.9	0.1	0.1	0.2	0.1	0.5

So, by Definition 3, we can define fuzzy bag $\tilde{\mathcal{B}}^{\tilde{f}} = (\tilde{f}, B^{\tilde{f}})$ where,

$$\tilde{f}(\text{young}) = \{\frac{0.7}{\text{Ben}}, \frac{0.2}{\text{Sue}}, \frac{0.4}{\text{Tom}}, \frac{0.0}{\text{John}}, \frac{0.7}{\text{Stan}}, \frac{0.4}{\text{Bill}}, \frac{0.2}{\text{Kim}}, \frac{0.7}{\text{Ana}}, \frac{0.1}{\text{Sara}}\},$$
$$\tilde{f}(\text{middle age}) = \{\frac{0.3}{\text{Ben}}, \frac{0.8}{\text{Sue}}, \frac{0.7}{\text{Tom}}, \frac{0.3}{\text{John}}, \frac{0.3}{\text{Stan}}, \frac{0.7}{\text{Bill}}, \frac{0.8}{\text{Kim}}, \frac{0.3}{\text{Ana}}, \frac{0.5}{\text{Sara}}\},$$
$$\tilde{f}(\text{old}) = \{\frac{0.1}{\text{Ben}}, \frac{0.2}{\text{Sue}}, \frac{0.1}{\text{Tom}}, \frac{0.9}{\text{John}}, \frac{0.1}{\text{Stan}}, \frac{0.1}{\text{Bill}}, \frac{0.2}{\text{Kim}}, \frac{0.1}{\text{Ana}}, \frac{0.5}{\text{Sara}}\},$$

and

$$\begin{aligned} B^{\tilde{f}} = \{&(\text{young}, 0.7, 3), (\text{young}, 0.4, 2), (\text{young}, 0.2, 2), (\text{young}, 0.1, 1), (\text{young}, 0.0, 1), \\ &(\text{middle age}, 0.8, 2), (\text{middle age}, 0.7, 2), (\text{middle age}, 0.5, 1), (\text{middle age}, 0.3, 4), \\ &(\text{old}, 0.9, 1), (\text{old}, 0.5, 1), (\text{old}, 0.2, 2), (\text{old}, 0.1, 5)\}. \end{aligned}$$

Remark 1. As it can be seen, the more important part of an fuzzy bag is information function $\tilde{f}$. Therefore, it is possible to study the properties of fuzzy bags just by considering their information functions.

Notation 2. We set $\tilde{\mathbf{B}}(P,O)$ as the set of all fuzzy bags $\tilde{\mathcal{B}}^{\tilde{f}} = (\tilde{f}, B^{\tilde{f}})$. Where, $\tilde{f}: P \to \mathcal{F}(O)$ and $B^{\tilde{f}}$ are as defined in Definition 3. Clearly, $\mathbf{B}(P,O) \subseteq \tilde{\mathbf{B}}(P,O)$.

Here, we can define intersection and union of fuzzy bags.

Definition 4. Let $\tilde{\mathcal{B}}^{\tilde{f}_i} \in \tilde{\mathbf{B}}(P_i,O_i)$ for all $i \in I_n$ be given fuzzy bags and $\overline{O} = \cup_{i\in I_n} O_i$. Then, their intersection is fuzzy bag

$$\cap_{i\in I_n}\tilde{\mathcal{B}}^{\tilde{f}_i} = (\cap_{i\in I_n}\tilde{f}_i, B^{\cap_{i\in I_n}\tilde{f}_i}).$$

Where, $\cap_{i\in I_n}\tilde{f}_i : \Pi_{i\in I_n}P_i \to \mathcal{F}(\overline{O})$ such that $(\cap_{i\in I_n}\tilde{f}_i)(p_1,p_2,\ldots,p_n) = \cap_{i\in I_n}\tilde{f}_i(p_i)$ for all $p_i \in P_i$.

Note that by Definition 3, $\cap_{i\in I_n}\tilde{\mathcal{B}}^{\tilde{f}_i} = \tilde{\mathcal{B}}^{\cap_{i\in I_n}\tilde{f}_i}$, where

$$B^{\cap_{i\in I_n}\tilde{f}_i} = \{((p_1,p_2,\ldots,p_n),\delta, card(O_\delta^{p_1,p_2,\ldots,p_n}))|p_i \in P_i, \delta \in [0,1]\},$$

where $O_\delta^{p_1,p_2,\ldots,p_n} = \{o \in \overline{O}|(\cap_{i\in I_n}\tilde{f}_i)(p_1,p_2,\ldots,p_n)(o) = \delta\}$.

Definition 5. Let $\tilde{\mathcal{B}}^{\tilde{f}_i} \in \tilde{\mathbf{B}}(P_i,O_i)$ for all $i \in I_n$ be given fuzzy bags and $\overline{O} = \cup_{i\in I_n} O_i$. Then, their union is fuzzy bag

$$\cup_{i\in I_n}\tilde{\mathcal{B}}^{\tilde{f}_i} = (\cup_{i\in I_n}\tilde{f}_i, B^{\cup_{i\in I_n}\tilde{f}_i}),$$

where $\cup_{i\in I_n}\tilde{f}_i : \Pi_{i\in I_n}P_i \to \mathcal{F}(\overline{O})$ such that $(\cup_{i\in I_n}\tilde{f}_i)(p_1,p_2,\ldots,p_n) = \cup_{i\in I_n}\tilde{f}_i(p_i)$ for all $p_i \in P_i$.

Note that by Definition 3, $\cup_{i\in I_n}\tilde{\mathcal{B}}^{\tilde{f}_i} = \tilde{\mathcal{B}}^{\cup_{i\in I_n}\tilde{f}_i}$, where

$$B^{\cup_{i\in I_n}\tilde{f}_i} = \{((p_1,p_2,\ldots,p_n),\delta, card(O_\delta^{p_1,p_2,\ldots,p_n}))|p_i \in P_i, \delta \in [0,1]\},$$

where $O_\delta^{p_1,p_2,\ldots,p_n} = \{o \in \overline{O}|(\cup_{i\in I_n}\tilde{f}_i)(p_1,p_2,\ldots,p_n)(o) = \delta\}$.

Definition 6. A fuzzy bag $\tilde{\mathcal{B}}^{\tilde{f}}$ is a fuzzy sub bag of $\tilde{\mathcal{B}}^{\tilde{g}}$, denoted by $\tilde{\mathcal{B}}^{\tilde{f}} \tilde{\sqsubseteq} \tilde{\mathcal{B}}^{\tilde{g}}$ if and only if $\tilde{f}(p) \tilde{\subseteq} \tilde{g}(p)$ for all $p \in P$. That means $\tilde{\mathcal{B}}^{\tilde{f}} \tilde{\sqsubseteq} \tilde{\mathcal{B}}^{\tilde{g}}$ if and only if for all $p \in P$, $\tilde{f}(p)$ be a fuzzy subset of $\tilde{g}(p)$.

Definition 7. Two fuzzy bags $\tilde{\mathcal{B}}^{\tilde{f}}$ and $\tilde{\mathcal{B}}^{\tilde{g}}$ are equal, denoted by $\tilde{\mathcal{B}}^{\tilde{f}} \cong \tilde{\mathcal{B}}^{\tilde{g}}$ if $\tilde{\mathcal{B}}^{\tilde{f}} \tilde{\sqsubseteq} \tilde{\mathcal{B}}^{\tilde{g}}$ and $\tilde{\mathcal{B}}^{\tilde{g}} \tilde{\sqsubseteq} \tilde{\mathcal{B}}^{\tilde{f}}$ that means if $\tilde{f} = \tilde{g}$.

The next theorem gives some useful results about fuzzy bags.

Theorem 1. *Operations* $\cup$ *and* $\cap$ *in* $\tilde{\mathbf{B}}(P,O)$ *satisfy the laws of idempotency, commutativity, associativity and distributivity.*

In the following definition, we introduce the concept of complement of a fuzzy bag.

Definition 8. Let $\eta : [0,1] \to [0,1]$ be a fixed strong negation [4], this means an involutive decreasing bijection. Consider $\tilde{\mathcal{B}}^{\tilde{f}} = (\tilde{f}, B^{\tilde{f}})$. Then, the $\eta-$complement of $\tilde{\mathcal{B}}^{\tilde{f}}$ is fuzzy bag $(\tilde{\mathcal{B}}^{\tilde{f}})^c = (\tilde{f}^c, B^{\tilde{f}^c})$, where $\tilde{f}^c : P \to \mathcal{F}(O)$ such that $\tilde{f}^c(p)(o) = \eta(\tilde{f}(p)(o))$ for all $p \in P$ and $o \in O$.

Note that by Definition 3, $(\tilde{\mathcal{B}}^{\tilde{f}})^c = \tilde{\mathcal{B}}^{\tilde{f}^c}$.

Note 1. In Definition 8, if η is the standard negation, $\eta(x) = 1 - x$ for all $x \in [0,1]$ [4], then $\tilde{\mathcal{B}}^{\tilde{f}^c}$ is called complement of $\tilde{\mathcal{B}}^{\tilde{f}}$.

Example 3. Consider the fuzzy bag of Example 2. The complement of this fuzzy bag is $\tilde{\mathcal{B}}^{\tilde{f}^c} = (\tilde{f}^c, B^{\tilde{f}^c})$ where,

$$\tilde{f}^c(\text{young}) = \{\frac{0.3}{\text{Ben}}, \frac{0.8}{\text{Sue}}, \frac{0.6}{\text{Tom}}, \frac{1.0}{\text{John}}, \frac{0.3}{\text{Stan}}, \frac{0.6}{\text{Bill}}, \frac{0.8}{\text{Kim}}, \frac{0.3}{\text{Ana}}, \frac{0.9}{\text{Sara}}\},$$
$$\tilde{f}^c(\text{middle age}) = \{\frac{0.7}{\text{Ben}}, \frac{0.2}{\text{Sue}}, \frac{0.3}{\text{Tom}}, \frac{0.7}{\text{John}}, \frac{0.7}{\text{Stan}}, \frac{0.3}{\text{Bill}}, \frac{0.2}{\text{Kim}}, \frac{0.7}{\text{Ana}}, \frac{0.5}{\text{Sara}}\},$$
$$\tilde{f}^c(\text{old}) = \{\frac{0.9}{\text{Ben}}, \frac{0.8}{\text{Sue}}, \frac{0.9}{\text{Tom}}, \frac{0.1}{\text{John}}, \frac{0.9}{\text{Stan}}, \frac{0.9}{\text{Bill}}, \frac{0.8}{\text{Kim}}, \frac{0.9}{\text{Ana}}, \frac{0.5}{\text{Sara}}\},$$

and

$$\begin{aligned} B^{\tilde{f}^c} = \{&(\text{young}, 1.0, 1), (\text{young}, 0.9, 1), (\text{young}, 0.8, 2), (\text{young}, 0.6, 2), (\text{young}, 0.3, 3), \\ &(\text{middle age}, 0.7, 4), (\text{middle age}, 0.5, 1), (\text{middle age}, 0.3, 2), (\text{middle age}, 0.2, 2), \\ &(\text{old}, 0.9, 5), (\text{old}, 0.8, 2), (\text{old}, 0.5, 1), (\text{old}, 0.1, 1)\}. \end{aligned}$$

4 Alpha-Cuts of Fuzzy Bags

The notion of α-cut plays a fairly big role in the fuzzy theory. So, here, we define this notion for the fuzzy bags.

Definition 9. Let $\alpha \in [0, 1]$. Then, α-cut of fuzzy bag $\tilde{\mathcal{B}}^{\tilde{f}} \in \tilde{\mathbf{B}}(P, O)$ is the crisp bag $(\tilde{\mathcal{B}}^{\tilde{f}})_\alpha = (\tilde{f}_\alpha, B^{\tilde{f}_\alpha})$ where, $\tilde{f}_\alpha : P \to \mathcal{P}(O)$ is a function in which for all $p \in P$, $\tilde{f}_\alpha(p) = \{o \in O | \tilde{f}(p)(o) \geqslant \alpha\}$ and

$$B^{\tilde{f}_\alpha} = \{(p, card(\tilde{f}_\alpha(p))) | p \in P\}.$$

Definition 10. Let $\alpha \in [0, 1]$. Then, strong α-cut of fuzzy bag $\tilde{\mathcal{B}}^{\tilde{f}} \in \tilde{\mathbf{B}}(P, O)$ is the crisp bag $(\tilde{\mathcal{B}}^{\tilde{f}})_{\alpha^\cdot} = (\tilde{f}_{\alpha^\cdot}, B^{\tilde{f}_{\alpha^\cdot}})$ where, $\tilde{f}_{\alpha^\cdot} : P \to \mathcal{P}(O)$ is a function which for all $p \in P$, $\tilde{f}_{\alpha^\cdot}(p) = \{o \in O | \tilde{f}(p)(o) > \alpha\}$ and

$$B^{\tilde{f}_{\alpha^\cdot}} = \{(p, card(\tilde{f}_{\alpha^\cdot}(p))) | p \in P\}.$$

Note that by Definition 1, we have $\mathcal{B}^{\tilde{f}_\alpha} = (\tilde{\mathcal{B}}^{\tilde{f}})_\alpha$ and $\mathcal{B}^{\tilde{f}_{\alpha^\cdot}} = (\tilde{\mathcal{B}}^{\tilde{f}})_{\alpha^\cdot}$.

Notation 3. We set $\tilde{f}_{[\alpha,\beta)}(p) = \{o \in O | \alpha \le \tilde{f}(p)(o) < \beta\}$ and $\tilde{f}_{(\alpha,\beta]}(p) = \{o \in O | \alpha < \tilde{f}(p)(o) \le \beta\}$ for all $p \in P$.

Some useful results for the fuzzy bags are given in the next theorem.

Theorem 2. *Let $\tilde{\mathcal{B}}^{\tilde{f}}, \tilde{\mathcal{B}}^{\tilde{g}} \in \tilde{\mathbf{B}}(P, O)$, $\alpha, \beta \in [0, 1]$ and $\alpha \leqslant \beta$. Then,*

i) $\mathcal{B}^{\tilde{f}_{\beta^\cdot}} \tilde{\sqsubseteq} \mathcal{B}^{\tilde{f}_\beta} \tilde{\sqsubseteq} \mathcal{B}^{\tilde{f}_{\alpha^\cdot}} \tilde{\sqsubseteq} \mathcal{B}^{\tilde{f}_\alpha}$,

ii) $\mathcal{B}^{\tilde{f}_\alpha} = \mathcal{B}^{\tilde{f}_\beta}$ *if and only if* $\mathcal{B}^{\tilde{f}_{[\alpha,\beta)}} = \mathcal{B}^0$,

iii) $\mathcal{B}^{\tilde{f}_{\alpha^\cdot}} = \mathcal{B}^{\tilde{f}_{\beta^\cdot}}$ *if and only if* $\mathcal{B}^{\tilde{f}_{(\alpha,\beta]}} = \mathcal{B}^0$,

iv) $(\tilde{\mathcal{B}}^{\tilde{f}} \cup \tilde{\mathcal{B}}^{\tilde{g}})_\alpha = \mathcal{B}^{\tilde{f}_\alpha} \cup \mathcal{B}^{\tilde{g}_\alpha}$ *and* $(\tilde{\mathcal{B}}^{\tilde{f}} \cup \tilde{\mathcal{B}}^{\tilde{g}})_{\alpha^\cdot} = \mathcal{B}^{\tilde{f}_{\alpha^\cdot}} \cup \mathcal{B}^{\tilde{g}_{\alpha^\cdot}}$,

v) $(\tilde{\mathcal{B}}^{\tilde{f}} \cap \tilde{\mathcal{B}}^{\tilde{g}})_\alpha = \mathcal{B}^{\tilde{f}_\alpha} \cap \mathcal{B}^{\tilde{g}_\alpha}$ *and* $(\tilde{\mathcal{B}}^{\tilde{f}} \cap \tilde{\mathcal{B}}^{\tilde{g}})_{\alpha^\cdot} = \mathcal{B}^{\tilde{f}_{\alpha^\cdot}} \cap \mathcal{B}^{\tilde{g}_{\alpha^\cdot}}$.

In the following example, we compute α-cuts of a fuzzy bag.

Example 4. Consider the fuzzy bag of Example 2. We compute α-cuts, $\mathcal{B}^{\tilde{f}_\alpha} = (\tilde{f}_\alpha, B^{\tilde{f}_\alpha})$. Where, $\tilde{f}_\alpha(p)$ is presented in Table 3.

and $B^{\tilde{f}_\alpha}$ is as follows

$B^{\tilde{f}_0} = \{(\text{young}, 9), (\text{middle age}, 9), (\text{old}, 9)\}$, $\quad B^{\tilde{f}_{0.1}} = \{(\text{young}, 8), (\text{middle age}, 9), (\text{old}, 9)\}$

$B^{\tilde{f}_{0.2}} = \{(\text{young}, 7), (\text{middle age}, 9), (\text{old}, 4)\}$, $\quad B^{\tilde{f}_{0.3}} = \{(\text{young}, 5), (\text{middle age}, 9), (\text{old}, 2)\}$

$B^{\tilde{f}_{0.4}} = \{(\text{young}, 5), (\text{middle age}, 5), (\text{old}, 2)\}$, $\quad B^{\tilde{f}_{0.5}} = \{(\text{young}, 3), (\text{middle age}, 5), (\text{old}, 2)\}$

$B^{\tilde{f}_{0.7}} = \{(\text{young}, 3), (\text{middle age}, 4), (\text{old}, 1)\}$, $\quad B^{\tilde{f}_{0.8}} = \{(\text{middle age}, 2), (\text{old}, 1)\}, B^{\tilde{f}_{0.9}} = \{(\text{old}, 1)\}$.

Table 3: The values of $\tilde{f}_\alpha(p)$ for Example 4

α \ p	young	middle age	old
0.0	O	O	O
0.1	O $\setminus$ {John, Sara}	O	O
0.2	O $\setminus$ {John, Sara}	O	O $\setminus$ {Sue, John, Kim, Sara}
0.3	O $\setminus$ {Sue, John, Kim, Sara}	O	{John, Sara}
0.4	O $\setminus$ {Sue, John, Kim, Sara}	{Sue, Tom, Bill, Kim, Sara}	{John, Sara}
0.5	{Ben, Stan, Ana}	{Sue, Tom, Bill, Kim, Sara}	{John, Sara}
0.7	{Ben, Stan, Ana}	{Sue, Tom, Bill, Kim}	{John}
0.8	$\emptyset$	{Sue, Kim}	{John}
0.9	$\emptyset$	$\emptyset$	{John}

Definition 11. Let $\mathcal{B}^f \in \mathbf{B}(P,O)$ and $\alpha \in [0,1]$. We define fuzzy bag $\widetilde{\alpha\mathcal{B}^f} = \tilde{\mathcal{B}}^{\widetilde{\alpha f}} = (\widetilde{\alpha f}, B^{\widetilde{\alpha f}})$ where,

$$\widetilde{\alpha f}(p)(o) = \min(\alpha, \chi_{f(p)}(o)) = \alpha\chi_{f(p)}(o),$$

for all $o \in O$ and $p \in P$.

Theorem 3. i) *Let $\tilde{\mathcal{B}}^{\tilde{f}}$ be a fuzzy bag and let $\mathcal{B}^{\tilde{f}_\alpha}$ be α-cut of $\tilde{\mathcal{B}}^{\tilde{f}}$. Then,*

$$\tilde{\mathcal{B}}^{\tilde{f}} = \bigcup_{\alpha\in[0,1]} \widetilde{\alpha\mathcal{B}}^{\tilde{f}_\alpha}.$$

i) *Let $\tilde{\mathcal{B}}^{\tilde{f}}$ be a fuzzy bag and let $\mathcal{B}^{\tilde{f}_{\alpha\cdot}}$ be the strong α-cut of $\tilde{\mathcal{B}}^{\tilde{f}}$. Then,*

$$\tilde{\mathcal{B}}^{\tilde{f}} = \bigcup_{\alpha\in[0,1]} \widetilde{\alpha\mathcal{B}}^{\tilde{f}_{\alpha\cdot}}.$$

Theorem 4. *Let $\tilde{\mathcal{B}}^{\tilde{f}} \in \tilde{\boldsymbol{B}}(P,O)$ and $\{\mathcal{B}^{\tilde{g}_\alpha}|\alpha \in [0,1]\}$ is a class of elements of $\boldsymbol{B}(P,O)$ such that $\mathcal{B}^{\tilde{f}_{\alpha\cdot}} \sqsubseteq \mathcal{B}^{\tilde{g}_\alpha} \sqsubseteq \mathcal{B}^{\tilde{f}_\alpha}$. Then,*

$$\tilde{\mathcal{B}}^{\tilde{f}} = \bigcup_{\alpha\in[0,1]} \widetilde{\alpha\mathcal{B}}^{\tilde{g}_\alpha}.$$

Theorem 5. *Let $\{\mathcal{B}^{g_\alpha}|\alpha \in [0,1]\}$ is a class of elements of $\boldsymbol{B}(P,O)$. There exists $\tilde{\mathcal{B}}^{\tilde{f}} \in \tilde{\boldsymbol{B}}(P,O)$ such that for all $\alpha \in [0,1]$, $\mathcal{B}^{\tilde{f}_\alpha} = \mathcal{B}^{g_\alpha}$ if and only if for all $\alpha, \beta \in [0,1]$ such that $\alpha \leq \beta$, $\mathcal{B}^{g_\beta} \sqsubseteq \mathcal{B}^{g_\alpha}$ and $\mathcal{B}^{g_0} = \mathcal{B}^1$.*

5 Conclusion

Using Delgado et al.'s definition of bags, which is improved version of Yager's one, a new definition for fuzzy bags has been introduced. Also, a concept of the $\alpha-$cut of fuzzy bags has been studied.

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