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Contents

PROPERTIES OF COMMON CAUSE IN QUANTUM LOGIC

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Keywords: s-map, common cause, orthomodular lattice

In this study, we focus on a definition of a common cause in quantum logic, which was basically defined in a probability space by Rédei in (1997). We study the properties of a common cause in quantum logic by using an s-map. We compare the structure of a common cause in a probability space to the structure of a common cause in quantum logic. Finally, we show an example that explain the construction of a common cause.

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A STUDY ABOUT ANALYTIC HIERARCHY PROCESS (AHP) IMPLEMENTATIONS FOR INTERNAL PROCESSES AND SUPPLIER SELECTION PROCESS AT PROCUREMENT DEPARTMENT OF A UNIVERSITY IN TURKEY

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Analytical Hierarchy Process is an approach helps to choose the best one of the alternatives satisfying needs and requirements of the complex decisions. At?l?m University was selected for the study. It is in Ankara, capital of Turkey and has 367 academic personnel, 182 administrative personnel, and 5500 undergraduate/master/doctorate student. The university has 2 Graduate Schools with 24 masters and 2 doctorate programs and a "School of Foreign Languages" which

include the English Preparatory Department. Also it has distance education programs and e-MBA. At first step of the research an AHP study was applied to choose the best one of supplier selections. In the application speed, quality, price, proximity, and time criteria were taken into account. Secondly, AHP was used for determining for which criteria is important for internal processes by considering easy to do, in-complexity, easy to integrate, and time criteria.

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SELECTING THE HIGH - PERFORMING DEPARTMENTS WITHIN UNIVERSITIES APPLYING THE FUZZY ELECTRE METHOD

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Keywords: Fuzzy ELECTRE, Triangular Fuzzy Numbers (TFN), Multi criteria decision-making (MCDM), Multi-attribute decision-making (MADM), Measurement

In this study, the process of efficiency measurement is tackled with using multiple criteria decision-making (MCDM) processes. Multi criteria decision-making (MCDM) is a modeling and methodological tool for dealing with complex engineering problem. In fact alternatives' evaluation is the concept that largely depends on the experts and their expertise, which increase uncertainty in the decision-making process. This way the final ranking is resulted from an independent fuzzy system, which has considered the existing uncertainty in the evaluations. So the framework of the study is based on Fuzzy ELECTRE Method. In this paper, we proposed the High - Performing Departments within Universities selection system based on Fuzzy ELECTRE. The Fuzzy ELECTRE method adopted here uses Triangular Fuzzy Numbers (TFN). The Fuzzy ELECTRE is applied to evaluate the High - Performing Departments with the rating criteria.

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ON THE APPLICATION OF THE FUZZIFIED CHOQUET INTEGRAL TO MULTIPLE CRITERIA DECISION MAKING

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In this paper, we employ the fuzzified Choquet integral [1] in multiple criteria decision making. The first part of the paper deals with a role of Choquet integral [3] in models of multiple criteria evaluation. For the Partial Goals Method [2] we compare a weighted average, which is an aggregation operator based on an additive measure, with aggregation operators employing a generalised monotonous measure - a fuzzy measure. We specify general conditions for application of Choquet integral to the aggregation of partial evaluations. The second part of the paper deals with the fuzzified Choquet integral. Similarly to [1], two levels of fuzzification are studied. The first level works with partial fuzzy evaluations, which are very helpful in the evaluation practice (such as expert's evaluations with respect to qualitative criteria). In the second level we consider also the values of the fuzzy measure (i. e. the weights of the sets of partial goals) to be modelled by fuzzy numbers. We explain the use of the fuzzified fuzzy measure in evaluation models (for example, for the weights of the set of criteria, which are determined expertly) and propose an effective way of setting the fuzzified fuzzy measure on a set of partial goals. The third part of the paper is devoted to some illustrative examples of applying Choquet integral and its fuzzified versions to multiple criteria decision making problems, together with scenarios, in which the use of Choquet integral is not advisable.

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FUZZY PREFERENCE STRUCTURES BASED ON GENERATED IMPLICATIONS

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Keywords: generated implication, fuzzy preference structure, triple of generators, strict preference and implications

Fuzzy preference structure is a triple of fuzzy relations (P, I, J), where P, I and J are relations of strict preference, indifference and incomparability respectively. This triple can be obtained using (p, i, j) called triple of generators. For fuzzy implication I and negation n, mapping $I(n(x), y)$ satisfies conditions needed for generator of strict preference p. On the other hand, it is possible to define fuzzy implication using function of a single variable in a way similar to a continuous Archimedean t-norm. We have already studied some generated implications and their properties. The aim of this paper is to investigate fuzzy preference structures in which the generator of a strict preference is defined using generated implications.

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VOTING SYSTEMS UNDER THE SCRUTINY OF AGGREGATIONS OPERATORS

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Elections as it is largely known are producing many different results according to the voting system used and counting and tabulations of results provided. We can understand this procedure as a collective preference on a set of alternatives, which can be seen as a result obtained by usage of different aggregation operators. This could allow us at least to help eliminate the worst possible cases by using different voting systems.

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A NEAR EQUITABLE 2-PERSON CAKE CUTTING ALGORITHM

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Keywords: Cake cutting, Algorithm, Approximation

In the cake cutting problem, we want to divide a cake among n players in such a way that every player gets a 'fair' share of the cake according to his valuation. The usually considered fairness criteria are proportionality, envy-freeness and equitability. For two players, the well-known procedure 'I cut, you choose' satisfies the first two conditions. The surplus procedure, proposed in 2006 by Steven J. Brams, Michael A. Jones, and Christian Klamler, achieves a form of equitability called proportional equitability but revealing of players' valuation functions is needed.

We present a new cake - cutting algorithm for two players which is near equitable i. e. the values of pieces assigned to players differ by at most ?, where ? > 0 is a given small number. Our algorithm does not require that players' valuation functions are common knowledge.

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$S(2,1)$-LABELING OF GRAPHS WITH CYCLIC STRUCTURE

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r-$S(2, 1)$-labeling of a graph G is a vertex coloring by the elements of the cyclic group $\mathbb{Z}_r$ such that adjacent vertices are labeled with elements at least 2 apart in $\mathbb{Z}_r$ and vertices at distance 2 are labeled with distinct elements of $\mathbb{Z}_r$. The smallest r such that a graph G has an r-$S(2, 1)$-labeling is called the σ-number of G [3].

We consider $S(2, 1)$-labelings of infinite families of graphs which are constructed from isomorphic segments cyclically arranged in such a way that the rotation by one segment is an automorphism of the whole graph, in particular the family of prisms and the family of the Isaacs graphs [2].

To obtain $S(2, 1)$-labelings of such graphs, we determine all r-$S(2, 1)$-labelings of a suitable subgraph at first and then concatenate them to an r-$S(2, 1)$-labeling of the whole graph. We use various support techniques, for instance group actions, voltage graphs [1], adjacency matrices of the transition graph of the r-$S(2, 1)$-labelings of the subgraph and few others.

We are able to obtain following results:

Theorem Y. Let Y_m denote the prism of order $2m$. Then

$$\sigma(Y_m) = \left\{ \begin{array}{ll} 6 & \text{for } m \equiv 0 \pmod 3 \\ 8 & \text{for } m \in \{4, 5, 8, 11\} \\ 7 & \text{for } m \not\equiv 0 \pmod 3 \text{ and } m \notin \{4, 5, 8, 11\}. \end{array} \right.$$

Theorem J. Let J_m denote the Isaacs graph of order $4m$. Then

$$\sigma(J_m) = \left\{ \begin{array}{ll} 8 & \text{for } m \in \{3, 4, 5, 7, 8, 9, 11\} \\ 7 & \text{for } m \in \{6, 10\} \text{ or } m \geq 12. \end{array} \right.$$

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A CONFIDENCE INTERVAL FOR THE PROBABILITY DIFFERENCE OF OVERALL TREATMENT EFFECTS - SIMULATION STUDY

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One of the main aims of the meta-analysis of clinical trials is the determination of the effectivity of a new type of treatment. The effectivity is determined by the difference of the effectivity of a standard treatment and the new treatment. In the case of binary data the difference can be measured by a probability difference. This paper presents the construction of the confidence interval for the probability difference of overall treatment effects in the meta-analysis based on multicentre trials. For the construction of the confidence interval the procedures of Wimmer & Witkovsky (2004) and Kenward & Roger (1997) have been used. The second part of this paper is a simulation study which presents properties of the proposed confidence interval.

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EQ-LOGICS: LOGICS DEVELOPED ON THE BASIS OF FUZZY EQUALITY

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Mathematical logic has been for many years developed on the basis of implication as the main connective. In eighties, a new direction of the development has been initiated which is called equational logic [2, 6]. It is based on equality as the main connective. This brought an idea to develop fuzzy logic on the basis of fuzzy equality (equivalence) as the main connective as well. If we take residuated lattice as a truth values structure [3] then we are forced to interpret fuzzy equality (equivalence) by a derived operation. Another possibility is to take EQ-algebra [4] as the algebra of truth values, because the basic operation in it is the fuzzy equality. Consequently, fuzzy equality in EQ-logic has a natural interpretation in EQ-algebras while implication becomes a derived connective.

In this paper we continue the work on EQ-logics initiated in [5], both propositional as well as first order ones. The goal is to show a possible direction in the development of mathematical fuzzy logics, in which axioms are formed as identities. We focus on three types of propositional EQ-logics – basic, involutive and prelinear one and prove completeness of each of them.

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A DISTANCE-BASED EXTENSION OF THE MAJORITY JUDGEMENT VOTING SYSTEM

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Keywords: Majority Judgement, linguistic assessments, voting systems

Recently, Balinski and Laraki [1, 2] have introduced a new voting system called *Majority Judgement* (MJ). They propose that voters assess candidates by means of a set of linguistic terms as *Excellent*, *Very Good*, *Good*, *Acceptable*, *Poor* and *To Reject*. Then, they consider the lower median as collective assessment, and they provide a sequential tie-breaking process to rank order the candidates. The authors advocate MJ because of its good properties with respect to the classic Arrow scheme of preference aggregation. However, MJ has some drawbacks that have been denounced by several authors. In this paper we propose to consider a metric on the set of linguistic terms in order to generate a collective assessment for each candidate, just one that minimizes the sum of distances among the individual assessments and the collective one: the Fermat linguistic term. We also provide a sequential tie-breaking process in such a way that the voting procedure satisfies some interesting properties within the Social Choice approach.

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ON THE USE OF INTERVAL-VALUED FUZZY SETS IN STEREO MATCHING PROBLEM

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Keywords: Interval-valued Fuzzy Set, Interval-valued similarity, Stereo matching problem

Stereo matching problem [1] is a difficult problem in computer vision, however, its application in many real world problems such as automatic car driving, industrial machinery or robot navigation, converts it into an interesting topic. The difficulty of the problem arises from different points, for instance, occlusions, perspective distortions, different lighting intensities, reflections, shadows, repetitive patterns, sensory noise, etc.

Fuzzy measures have been employed within stereo matching problem [2] trying to overcome these issues. Our aim is to exploit the flexibility of Interval-Valued Fuzzy Sets to improve the matching ability of the algorithm by using a more intuitive representation of the images.

We present a stereo matching algorithm using Interval-valued Fuzzy Sets in order to improve the representation of the images. Then, we put forward the Interval-valued fuzzy similarity [3] to compare windows with the pixels represented by intervals. Finally, we compare the local matching measure accuracy of our algorithm with similar ones showing a promising behaviour.

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AVOIDING INCONSISTENCIES IN MAJORITY DECISIONS

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Keywords: Preferences, simple majority, acyclicity, voting paradox

In order to avoid some drawbacks of simple and absolute majorities, in [1] are introduced and analyzed *majorities based on difference of votes*: the alternative x is collectively preferred to the alternative y when the number of individuals who prefer x to y exceeds the number of individuals who prefer y to x by at least a fixed integer threshold k. In [2] the class of *majorities based on difference of support* extends the class of majorities based on difference of votes by allowing individuals to declare their intensities of preference among the alternatives by means of reciprocal preference relations and by using thresholds not necessarily integers. In this paper we have obtained some necessary and sufficient conditions for ensuring that the ordinary collective preferences associated with majorities based on difference of support are acyclic when voters show their graded preferences on three alternatives. In these results we have considered three different transitivity conditions for individual reciprocal preference relations.

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SOFTWARE FOR MULTIPLE CRITERIA EVALUATION SUPPORT - THE AGGREGATION OF PARTIAL EVALUATIONS BY CHOQUET INTEGRAL

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Keywords: Choquet integral, software.

The most of multiple criteria evaluation models use weighted average for the aggregation of partial evaluations. This type of aggregation requires additivity of the partial evaluations. If the condition of additivity is not satisfied, and if there are globally valid synergic or dyssynergic relations among the criteria, Choquet integral is suitable for the aggregation of partial evaluations. It makes sense to consider that the partial evaluations (the integrated function) and their significances (the fuzzy measure used in Choquet integral) are uncertain, i.e., fuzzy.

New software for multiple criteria evaluation by the Choquet integral was created. It is based on the theory published in [1] and on the generalization of the fuzzified Partial Goals Method [2, 3, 4]. Similarly like in [2], the evaluation models created in the frame of this software work with a special type of fuzzy evaluations - these evaluations represent the fuzzy degrees of fulfillment of the corresponding goals, or groups of goals. The significances of partial goals and their groups are described by a fuzzified fuzzy measure defined on the set of goals. The partial evaluations aggregation is realized by fuzzified Choquet integral.

The software is user-friendly. It helps user to choose the right aggregation operator by asking him/her simple questions on the characteristics of the decision-making problem. For solving the problem, following aggregation operators are available - weighted average, fuzzy MAXMIN, fuzzy MAXMAX, fuzzy OWA and the general fuzzy Choquet integral. A special care was devoted to the user interface for setting the weights of partial goals and their groups. Therefore, the weights can be set in intelligible and user-friendly way. The software guides the user through all steps of solving the decision-making problem and makes it easy to solve even complex problems.

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EXTREME-VALUE COPULAS

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Keywords: copula, extreme-value distribution, extreme-value copulas

Based on extreme - value distributions, extreme - value copulas are recalled. A complete description of 2-dimensional extreme - value copulas is given, including some construction methods for dependence functions. Observe that a function $C : [0,1]^2 \rightarrow [0,1]$ is an extreme - value copula if and only if $C(x,y) = (x,y)^{A(\frac{logx}{logxy})}$, where $A : [0,1] \rightarrow [0,1]$ is a dependence function, A is convex and $\max(x, 1-x) \leq A(x) \leq 1$. Extreme - value copulas of higher dimensions are discussed, too. Some new families of 3-dimesional extreme-value copulas are introduced.

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ALCORE - ARTIFICIAL LIFE SIMULATOR

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Keywords: Artificial life simulator, reactive agents, model application

Artificial life, known as alife as well, studies systems corresponding with life, its processes and evolution. With the help of computer models we study artificial life, evolution of its particular elements or entire population in the artificial created environment.

There are a lot of various tools in the field of computer artificial life simulation. There are tools which are available for particular tasks, or tools for more wide usage, which allows us to define some parameters of a simulation.

The aim of the paper is to sketch out how to develop a simulator based on reactive agents' principle, which should serve as an artificial life core for other applications. Its development was one of the topics of my diploma thesis [1]. The simulator's purpose is not to replace present simulators, but to reveal the basic principle of simulator development, from its design to particular application. We will be able to obtain some idea about basic structure of reactive agents based simulator. Main goals are speed, reusability, expandability and independent output.

Whole paper should have following content. At first, couple of present simulators will be introduced and we will emphasize on their application areas. Next, we will concentrate on proper simulator, ALCore, reason of its development, its features, evolution, usability and applicability. As conclusion, there will be its particular application presentation.

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Figure 1: Simulator ALCore - experiment with bees

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ON TWO CLASSES OF GENERATED COPULAS

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Keywords: copulas, horizontal generator, vertical generator

Based on notions introduced in [2], Durante and Jaworski [1] have recently introduced a special classes of copulas related to horizontal and vertical generators $f : [0,1] \to [0,\infty]$, f is continuous, convex, strictly decreasing and $f(1) = 0$ (observe that in the case of Archimedean copulas [3], such functions are called additive generators).

For a horizontal generator f, the copula $C_f : [0,1]^2 \to [0,1]$ is given by

$$C_f(x,y) = xf^{-1}(min(f(0), \tfrac{f(y)}{x})) \text{ if } x > 0.$$

For a vertical generator f, the copula $C^f : [0,1]^2 \to [0,1]$ is given by

$$C^f(x,y) = yf^{-1}(min(f(0), \tfrac{f(x)}{y})) \text{ if } y > 0.$$

In our contribution we introduce several classes of these generated copulas and discuss some of their properties.

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CONSTRUCTION OF INTERVAL-VALUED INTUITIONISTIC FUZZY SETS FROM FUZZY SETS. SOME OPERATORS

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Keywords: Interval-Valued Intuitionistic Fuzzy Sets, Fuzzy Sets, Operators

In this work we present a construction method of Interval-Valued Intuitionistic Fuzzy Sets ($IVIFSs$) [2] from Fuzzy Sets. For this purpose, we use a specific function g that represents the doubt in the assignement of the membership degree of every element to the Fuzzy Set. The main expression of our construction method is the following:

$$A \in IVIFSs(U) = \{< [\mu_{A_F}(u)(1 - \delta g(u)), \mu_{A_F}(u)(1 - \gamma g(u)) + \rho g(u)],$$
$$[(1 - \mu_{A_F}(u))(1 - \delta' g(u)), (1 - \mu_{A_F}(u))(1 - \gamma' g(u)) + \rho' g(u)] | u \in U >\}$$

being $A_F \in FSs(U)$, and $\delta, \delta', \gamma, \gamma', \rho, \rho' \in [0, 1]$.

We analize deeply every parameter in this construction and we present the relations they must fulfill.

We also present the opposite process, it means, how to obtain a Fuzzy Set from an IVIFS. For this reason, we study some operators to associate an IVIFS with an Atanassov's Intuitionistic Fuzzy Set (A-IFS) [1] and others to obtain a Fuzzy Set from an A-IFS.

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CENTRAL LIMIT THEOREM ON MV-ALGEBRAS

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Keywords: MV-algebra, Central limit theorem

The aim is to approve the Central limit theorem on MV-algebras by the new approach, using the observable as a distribution function, and not the σ-homomorphism. The main idea is in local representation of σ-algebras.

The following theorem is proved: Let M be a σ-complete MV-algebra with product, $m : M \longrightarrow [0, 1]$, be a σ- additive state, $(x_n)_n$ be a sequence of independent, equally distributed, square integrable observables. Then for any t $\in R$

$$\lim_{n \to \infty} m \left(\frac{\frac{1}{n} \sum_{i=1}^{n} x_i - E(x_i)}{\frac{\sigma}{\sqrt{n}}} ((-\infty, t)) \right) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{t} e^{-\frac{u^2}{2}} \, du$$

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ULTRAMODULAR AGGREGATION FUNCTIONS AND A NEW CONSTRUCTION METHOD FOR COPULAS

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Keywords: Copula, ultramodular function, aggregation function

Ultramodular aggregation functions are investigated and discussed, including a study of structural properties and the proposal of some construction methods. The weakest ultramodular aggregation function is the weakest aggregation function A_w, while there is no strongest ultramodular aggregation function. Each weighted arithmetic mean W_k given by $W_k(x, y) = kx + (1 - k)y$ with $k \in [0, 1]$ is a maximal element of the set of ultramodular aggregation functions.

So, special attention is paid to bivariate ultramodular aggregation functions, where modular functions and copulas play a substantial role, but also Archimedean ultramodular copulas are characterized and a new method for constructing bivariate copulas based on ultramodular aggregation functions is proposed and exemplified.

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RELATIONAL GALOIS CONNECTIONS IN THE FUZZY SETTING

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Keywords: Galois connections, formal concept analysis, fuzzy logic

Galois connections between two sets are the fundamental mappings in the formal concept analysis. Ganter in [1] sumarizes their generalization: Relational Galois connections between relations. We define analogic mappings in the fuzzy setting (studied is the case where structure of truth-degrees forms a complete residuated lattice) and study their properties. In addition, we investigate similar generalization based on an alternative of Galois connections—isotone Galois connections [2]. We express the relationship between the two generalizations; and between the fuzzy setting and the crisp setting. We show that antitone and isotone relational Galois connections between relations have similar properties. It turns out that they are related to fuzzy relational triangular products (studied by Bandler and Kohout) and compositions of both isotone and antitone Galois connections between sets.

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DISCRETE DUALITY FINITE VOLUME SCHEME FOR THE CURVATURE DRIVEN LEVEL SET EQUATION

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Keywords: curvature driven level set equation, discrete duality finite volume method

There are many approaches to obtain numerical solution of the curvature driven level set equation [4]. They are based either on finite differences method [4], or finite element method [2] or finite volume method [3]. Proposed numerical scheme uses so called discrete duality finite volumes like in [1].

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PARALLEL ALGORITHMS FOR FORMAL CONCEPT ANALYSIS

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Keywords: Formal Concept Analysis, Algorithms, Parallel, Distributed

Formal concept analysis (FCA) is a method of data analysis that enjoys particular interest in data-mining and information retrieval communities. It was proposed by Rudolf Wille [1] in eighties of the last century and since then it has evolved into a discipline having solid mathematical foundations, practical applications, and broad outreach to other analytical methods.

The input of the method is an object-attribute table, i.e., a table in which rows represent objects, columns represent their features and crosses and blanks (1's and 0's) in the table indicate that given object has a particular feature or not. The primary output of this method are maximal rectangular submatrices full of crosses present in the table. These submatrices, called *formal concepts*, correspond to natural concepts hidden in the data. Moreover, formal concepts form a hierarchy allowing to model natural subconcept-superconcept relationship.

In practice, the majority of applications of FCA depends on algorithms for computing all formal concepts for given input data table. It is therefore important to have fast and efficient algorithms for FCA, especially in case of processing large data.

One goal of our research is to propose fast and efficient algorithms for FCA that can be used to process large datasets.

Furthermore, current hardware development trends indicate that it can be expected that the demand for scalable parallel algorithms capable to utilize hardware of modern computers will grow.

In our contribution, we outline a family of novel parallel and distributed algorithms for FCA which are improving the Close-by-One algorithm [3] and allow to reduce time of data analysis, see [2]. Moreover, these algorithms are well-suited for parallel and distributed computers. Our contribution contains an introduction to FCA, description of the Close-by-One algorithm and its parallel and distributed variants, and performance evaluation of the algorithms.

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CONFLUENCE AND TERMINATION OF FUZZY RELATIONS: A SURVEY OF RESULTS

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Keywords: confluence, termination, fuzzy relation, similarity

Confluence and termination are essential properties of relations connected to the idea of rewriting and substituting which appear in abstract rewriting systems [1]. As shown in our previous publications [2, 3], confluence, termination, and related notions can be introduced as properties of fuzzy relations [4], leaving the ordinary notions a particular case when the underlying structure of truth degrees is the two-valued Boolean algebra. This contribution presents also a preliminary study of confluence and termination of fuzzy relations on similarity spaces [4]. The main motivation for this approach is the fact that, in many cases, the universe of discourse is equipped with a similarity relation. Interestingly, using a *reductionist approach* we will show that the newly introduced notions can be reduced to the notions defined in [2, 3].

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ON POSITIONED ECO-GRAMMAR SYSTEMS

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Positioned eco-grammar systems (PEG systems, for short) were introduced in Langer, M.: Agents placed in the environment of eco-grammar systems - Positioned eco-grammar systems. In: *Pre-Procs. of the 1st Doctoral Workshop on Mathematical a Engineering Methods in Computer Science* (M. Češka et al., Eds.), 2005. Motivation of introducing positioned eco-grammar systems is an attempt to describe interplay between evolving environment and community of agents living in this environment whereas we focus on embodiment of the agents and their presence in the environment. The environment is represented by 0L scheme which is a paralel rewriting system inspired by multicellular organisms (see Păun, et al.: Families Generated by Grammars and L Systems. In: *Handbook of Formal Languages Vol.1*, 1997). Agents are placed within the environment. Presence of each agent in the environment is given by its identifier. Agent changes the environment by rules of type $a[1] \rightarrow \alpha$ or $[1]a \rightarrow \alpha$, where $[1]$ is agent identifier, a are symbols from the environment and α is a string containing an arbitrary number of agents identifiers and environment symbols in arbitrary order. PEG systems bring a new view to investigating interplay between community of agents and environment. Our approach allows studying local changes in evolving environment caused by agents moreover the position of the agent is strictly given by special symbol and we are able to predict its behavior and control the evolution of environment as well. Biological motivation for introducing this kind of model can be found whenever we have evolving environment without outer influences, for example some biological material in Petri dish, and community of agents acting in such that environment, for example some viruses or bacteria.

In this paper we compare PEG systems with pure (regulated) grammars. We deal with (programmed) pure context free grammars without appearance checking and with pure grammars of type 0 (see Dassow, et al.: *Regulated Rewriting in Formal Language Theory*, 1986.). We show that family of languages generated by pure programmed context free grammars without appearance checking is a subset of family of PEG languages, and family of languages generated by pure context free grammars is a proper subset of family of PEG languages. We also show that family of languages generated by pure grammars of type 0 is a subset of family of PEG languages. Even though this statement includes both previous results, we give the proofs because of technique of the proof.

COMPARISON OF NUMERICAL AND ANALYTICAL APPROXIMATIONS OF THE EARLY EXERCISE BOUNDARY OF THE AMERICAN PUT OPTION

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Keywords: option pricing, American put option, early exercise boundary, limiting behavior close to expiry

In this paper we present qualitative and quantitative comparison of various analytical and numerical approximation methods for calculating a position of the early exercise boundary of the American put option paying zero dividends.

First we analyze their asymptotic behavior close to expiration. In the second part of the paper, we introduce a new numerical scheme for computing the entire early exercise boundary. The local iterative numerical scheme is based on a solution to a nonlinear integral equation. We compare numerical results obtained by the new method to those of the projected successive over relaxation (PSOR) method and the analytical approximation formula recently derived by Zhu [5]. In short-term horizon, we compared also asymptotic formulae by Evans, Kuske and Keller [2, 1], Stamicar, Ševčovič and Chadam [4].

To sum up results we obtained, analytical approximation formulae gives us good approximation of the free boundary in time close to expiration time. We discovered that Zhu's formula has different asymptotics and thus it slightly undershoots early exercise boundary close to expiration. On the other hand, in the long-term horizon, both Zhu's formula and new numerical methods gives us very good approximation of boundary in comparison with PSOR methods. To conclude, proposed new numerical scheme can be succesfully used for calculation of early exercise boundary.

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TESTING OF MARKOV ASSUMPTIONS BASED ON THE DYNAMIC SPECIFICATION TEST

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Keywords: Markov-switching model, Markov assumptions, dynamic specification test, testing non-linearity, testing remaining non-linearity

In this paper we discuss an alternative approach to testing linearity against Markov-switching type non-linearity. We aim to avoid the classic testing via the likelihood ratio test, which doesn't have a standard distribution. That's why time-consuming simulations must be carried out. We suggest the classical test to be substituted by using Hamilton's dynamic specification test for validity of Markov assumptions. The same idea will be applied to testing the remaining non-linearity to compare 2-regime with 3-regime models. These two approaches will be confronted by being demonstrated on 100 selected real time series.

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VALUED FORMAL CONCEPT ANALYSIS

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Keywords: Formal concept analysis, data mining, OLAP

The Formal Concept Analysis (FCA) is well-known method of the data mining. This method has very strong and solid mathematical background based on Galois connections and lattice theory. Another data mining method - Online Analytical Processing (OLAP), has trivial mathematical background (primarily based on simple aggregation functions), but has very strong technical reasons, why is used in many areas of data analysis (primarily in financial analysis). This paper is trying to put together strong mathematical background of FCA and easy, but powerful idea of OLAP, and is trying to introduce new data mining method - Valued Formal Concept Analysis - VFCA. At the beginning there are some mathematical definitions of FCA and VFCA, then some proofs of VFCA lattices properties. There is also described method of VFCA dealing with attribute implication and association rules. One of the problem of FCA is too much data on output. There are couple of approaches, how to set the constraint to output data, to obtain reasonable amount. One of the approach is based on closure operators, which has interesting mathematical properties. Approach is described for FCA already and in this paper will be described also approach for VFCA, which contains specific issues. At the end of the paper will be introduced web based prototype of VFCA application with couple of examples.

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ON HELMHOLTZ-LIKE FORMS

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In the talk, Helmholtz mapping $\mathcal{E}_2 : \Omega_2^r/\Theta_2^r \to \Omega_3^r/\Theta_3^r$ which assigns to every dynamical form E a Helmholtz form H_E is investigated. E represents a system of differential equations

$$E_i(t, x^k, \dot{x}^k, \ddot{x}^k) = 0, \quad 1 \leq i \leq m, \tag{1}$$

that need not be variational. It is well known that equations (1) are variational if and only if E_i's satisfy the Helmholtz conditions. The Helmholtz conditions describe the kernel of the morphism $\mathcal{E}_2$. We study the image of $\mathcal{E}_2$ and the kernel of the next mapping (morphism) $\mathcal{E}_3$. We solve the corresponding inverse problem when a 3-form corresponds to a system of differential equations. Our results extend known results on Lagrangians and locally variational dynamical forms to general dynamical forms, and open a new possibility to study non-variational equations by means of closed 3-forms as a parallel to extremal problems that are studied by means of closed 2-forms.

A STUDY OF THE RELATIVE EFFICIENCY OF SUPERMARKETS; A PILOT APPLICATION IN TURKEY USING DATA ENVELOPMENT ANALYSIS

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Keywords: envelopment analysis (DEA), supermarkets productivity, departmental productivity

Data envelopment analysis (DEA) is an application of linear programming that has been used to measure the relative efficiency of operating units with the same goals and objectives. In this paper, the productivity and performance of a retail company, a joint-stock company called Şeref Makromarket A.Ş, was evaluated. It competes in retail in Turkey. It has totally 3600 personnel and 113 supermarkets in 9 provinces of Turkey. In fact, the research focuses 74 supermarkets in the capital of Turkey, Ankara. In this research, the number of orders/deliveries, the number of personnel, the number of customers (receipts) per square meter, total number of customers, the number of customer complaints, annual income, inventory turnover rate, and annual cost including personnel, backorder, and inventory cost only were taken as input and financial profitability outputs, sales and profit, were firstly

investigated according to the inputs. After that, average productivity value of the company and both financial profitability and the average value were investigated respectively by applying the same study. In the second step of the study, a DEA model was developed for evaluating productivity of departments of the company including marketing, advertisement and public relations, logistics, and procurement only. Finally, evaluations and recommendations were presented by considering scores and results obtained by the DEA models and inputs.

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DYNAMIC MODEL OF A SMALL OPEN ECONOMY UNDER FLEXIBLE EXCHANGE RATES

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Keywords: Dynamical model, equilibrium point, matrix of linear approximation, eigenvalues, bifurcation equation, business cycle

In the paper a three dimensional dynamic model of a small open economy, describing the development of net real national income, real physical capital stock and expected exchange rate of near future is analyzed under flexible exchange rates. Sufficient conditions for the existence of a pair of purely imaginary eigenvalues with the third one negative in the linear approximation matrix of the model are found. Formulae for the calculation of the bifurcation coefficients in the bifurcation equation of the model are derived. Theorem on the existence of business cycles is presented.

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ON THE PARAMETERIZATION OF BADDELEY'S ERROR METRIC

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Keywords: Baddeley error metric, Binary image comparison, Edge detection

Baddeley [1] proposed a method for comparing binary images, inspired by the Hausdorff distance. This metric is based on the replacement of the supremum of the inter-set distances by their p-th order mean. However, the metric is dependent on three different parameters, whose impact has not been studied before.

In this work we propose the analysis of the parameterization of the metric, focusing on the application of comparing binary edge images.

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AN APPLICATION ABOUT PERFORMANCE OF AIR CARGO CARRIERS VIA DEA AND AHP METHODS

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Keywords: Data Envelope Analysis (DEA), Analytical Hierarchy Process (AHP), International Air Transport Association (IATA), Air Cargo Carriers, freight tones carried (FRT)

In this study air cargo operations performance was evaluated by the carriers' point of view. The objective of this paper is to evaluate the efficiencies of all-cargo carriers in terms of physical values and making a hierarchy between them in air cargo market via DEA/AHP method. Eight all-cargo carriers were determined from different parts of the world according to regional classification of IATA (International Air Transport Association). Two of them are from Asia, three of them from Europe, one of them from North America, and remaining two from Middle East. Three input variables and one output variable were determined. The input

variables are the number of yearly airline personnel, fleet (the number of aircraft in service yearly, including leased aircrafts from other airlines, excluding aircraft leased out to other operators), and stage length (the ratio of total aircraft kilometers from revenue flights to number of aircraft departures). The output variable is freight tones carried (FRT). All variables are taken from the carrier companies stated above with a single exception is the stage length. Stage length is calculated via the ratio of total aircraft kilometers from revenue flights to number of aircraft departures.

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SOLUTION OF APPLIED TASKS WITH PARALLEL PROGRAMMING

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Keywords: inverse method, conjugate gradient method

This context mathematic modeling *tells* about decision of inverse problem, which consists of cardinal estimation of model's parameters according to available information.

The common approach is to find such parameters of models on which specified functional which describes difference among calculated on models and measured quantity which reaches to minimum in space of parameters. However, in consequence of Adamar's tactlessness, peculiar to inverse problem, minimization of such functional not always connects with decrease errors in decision. Because of tactlessness, separate actual task will be the building of stable methods of inverse problem's decision in computer.

Purpose of my research work is creation and justification of methods of numerical founding of models' of thermal conduction's coefficients by data of boundary dimension of diffusion process's characteristics in flaky mediums.

Research methods:

The main results of research work were gotten by using application methods of mathematical modeling, theory of inverse problem and uncouth problem, theory differential calculus, functional analysis of calculus mathematics. To get analysis of formulas were used the computer algebra's system. To increase the effectiveness of program realization were used methods of parallel programming.

Main results:

It was specified the features of gradient algorithm decision differential analogue of inverse problem.

It was developed numerical algorithm for research and decision of differential inverse problem on the basis of combined using of space local-conjugate tasks.

By using parallelization with OpenMP, it was created complex program, which realizes the decision's algorithm and research of differential inverse method.

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ALGORITHM FOR APPROXIMATE DECOMPOSITION OF BOOLEAN MATRICES USING THEIR CONCEPTUAL STRUCTURE

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Keywords: boolean matrix, formal concept analysis, approximate methods

Boolean matrix decomposition is an important topic in relation data analysis. It aims to decompose the matrix I describing relationship between objects (rows) and attributes (columns) into two matrices A and B such that multiplication of A and B gives again I. The decomposition reveals a third entity playing role in the object-attribute relation, a set of factors. Then A represents relation between objects and factors, and B relation between factors and attributes.

An suitable measure of optimality of decomposition process is the minimal number of factors. We can get optimal decomposition with formal concepts as factors using a two step approach to the process: first compute the set of formal concepts, and then select the concepts that form the decomposition.

The goal of our work is to study aproximate decomposition, that is the decomposition of I into matrices A and B in a way that $A \cdot B$ gives a matrix approximately equal to I. Our motivation lies in the fact that we can get lower number of factors while the error causing the approximity is low and reasonable wrt. conceptual structure of I. We present novel heuristic algorithms and their comparison with existing methods.

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CONSTRUCTION OF ATANASSOV'S INTUITIONISTIC FUZZY SETS FROM FUZZY SETS. SOME OPERATORS

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The purpose of this work is to study a construction method of Atanassov's intuitionistic fuzzy sets [1] from fuzzy sets and a function $g : [0,1] \to [0,1]$ that satisfies the following conditions:

1. $g(x) = g(1-x)$;

2. If $x = 0.5$ then $g(x) = 1$;

3. $g(x) = 0$ if and only if $x = 0$ or $x = 1$.

For us, the function g represents the uncertainty of the expert when he assigns the membership degree of an element to a fuzzy set.

Besides, we study two operators that associate a fuzzy set with an Atanassov's intuitionistic fuzzy set: the K_α operator [2] and the $P_\mathbf{x}$ operator. We analyze their properties and some construction methods.

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GRAPHICAL SUPPORT OF SIMILARITY BASED FUZZY INTERPOLATION

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Keywords: fuzzy interpolation, similarity relation, distance

Our main idea proceeds from a theory described in [2] where the fuzzy interpolation is based on similarity relations [3]. From our observations we can say that the method presented in [2] solves an approximation problem instead of interpolation [1]. We focused on triangular fuzzy sets and we found out that the solution proposed in [2] does not always depend on an input fuzzy set. We also studied compositions between similarity relations and fuzzy sets and we discovered some properties of these compositions. In our approach we propose to rewrite the technique in [2] using metric distances and demonstrate all steps in their graphical representation.

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MODELLING OF DEPENDENCE STRUCTURE OF THE REGIME-SWITCHING MODELS RESIDUALS USING AUTOCOPULAS

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Keywords: time series, regime-switching models, residuals, autocopula, product copula, goodness-of-fit tests for autocopulas

The autocorrelation function is not suitable for description of residual dependence of regime-switching models. Therefore we want to investigate description of this dependence with 'k-lag auto-copula', which is 2-dimensional joint distribution function of the bivariate random vector (Y_t, Y_{t-k}) of time lagged values of random variables that generate time series (in the analogy of the autocorrelation function of stationary time series). In this contribution we will describe the dependence of time lagged residuals of SETAR models by means of copulas and we will test the independence of these residuals.

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VISUAL CHARACTERISATION OF ASSOCIATIVITY OF TRIANGULAR NORMS

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Keywords: Triangular norm, associativity, web geometry, Reidemeister closure condition, fuzzy logic

When dealing with triangular norms [1], the commutativity, the non-decreasingness and the existence of a neutral element have an easy graphical interpretation. However, the question how to visually interpret the associativity is a long-standing open problem within the community of people dealing with these operations which play a crucial role in fuzzy logic as logical conjunctions. Web geometry [2], which is a branch of differential geometry, has come with results which answer such, and similar, kinds of questions in a surprisingly intuitive and transparent geometric way. In particular, associative loops are characterized by the Reidemeister closure condition [3]. These results were, however, done to characterize algebraic properties of loops. Although triangular norms do not form loops, we show that the Reidemeister closure condition can be adopted in order to visually characterize associativity of triangular norms.

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CONSTRAINED PROBLEMS SOLVED BY COMPETITIVE DIFFERENTIAL EVOLUTION

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The differential evolution (DE) algorithm is one of the most frequently used algorithms for solving global optimization problems. The competition in differential evolution was shown to be highly effective way to avoid time-consuming process of setting most of the parameters of the algorithm. Several modifications of competitive differential evolution algorithm for single objective constrained real-parameter optimization problems are proposed. New algorithms are tested and compared on the family of problems developed for the special session of CEC2010.

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FINITE ELEMENT METHOD ON SURFACES

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In this paper, we discuss a solution of elliptic and parabolic equations on surfaces by the finite element method (FEM). We will present a FEM algorithm for solving a Laplace-Beltrami equation on a 3D surface and its extension to the case of surfaces evolving by mean curvature. We encompass numerical examples of elliptic and parabolic problems posed on open and closed surfaces embedded in 3D space.

LINGUISTIC FUZZY CONTROL OF MAGNETIC LEVITATION

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Fuzzy control has been motivated by employing an expert knowledge formulated in natural language. In this paper we suppose that the expert knowledge is expressed using fuzzy/linguistic IF-THEN rules

$$\text{IF } X_1 \text{ is } \mathcal{A}_1 \text{ AND } \cdots \text{ AND } X_n \text{ is } \mathcal{A}_n \text{ THEN } Y \text{ is } \mathcal{B}$$

where $\mathcal{A}_1, \ldots, \mathcal{A}_n, \mathcal{B}$ are so called *evaluative linguistic expressions* [1]. A set of such rules is called a *linguistic description*. A conclusion on the basis of the latter is obtained using a special inference method called *perception based logical deduction* [2]. Linguistic description characterizes appropriate control strategy. It can be learned from data generated on the basis of manual control. Some processes, however, cannot be controlled manually. Such a process is magnetic levitation. In this paper, we present linguistic control of the magnetic levitation on the basis of the control strategy obtained from an expert in the form of a linguistic description. In the paper, we will discuss our experiences, advantages and problems which occurred during development of the linguistic controller.

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PERSONNEL SELECTION USING FUZZY ANALYTIC HIERARCHY PROCESS (FAHP): AN APPLICATION

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Keywords: Academic Staff Selection, Multiple Criteria Analysis, Analytic Hierarchy Process, Fuzzy Analytic Hierarchy Process

There are various methods regarding personnel selection in the literature. Due to the increasing education improvement, university in the world demand to have quality and professional academic staff. In this paper, it is proposed an academic staff selection system based on Fuzzy Analytic Hierarchy Process (FAHP). The FAHP is applied to evaluate the best academic staff with the rating criteria. The result obtained by FAHP is compared with result produced by Mikhailov weighted goals method. The FAHP method adopted here uses Triangular Fuzzy Numbers (TFN).

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USING SIMILARITY MEASURES IN FUZZY RULE-BASED CLASSIFICATION SYSTEMS WITH INTERVAL-VALUED FUZZY SETS

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Fuzzy Rule-Based Classification Systems (FRBCSs) are a widely used tool in Data Mining because of the interpretability given by the concept of linguistic label. However, the use of this kind of models implies a degree of uncertainty in the definition of the fuzzy partitions. In this work we will use the concept of Interval-Valued Fuzzy Set (IVFS) to deal with the posible uncertainty . In [4], we have

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proved that the use of IVFSs is useful in the framework of classification with imbalanced data-sets.

In this contribution we generate the initial knowledge base employing the well-known rule learning algorithm given by Chi et al. [1]. We build the IVFSs starting from the fuzzy sets created by this method. In this way, we study the influence of IVFS in the Fuzzy Reasoning Method (FRM), not in the rule learning process.

We modify the FRM in the following way: we compute the matching degree between the rule antecedent and the example by means of similarity measures. The remaining of the FRM is kept as in the original one. We use the similarity measures given by Bustince [2] and Mendel [3] and we compare their respective results among them and with respect to the initial FRBCS.

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A FUZZY MODEL OF ACADEMIC STAFF EVALUATION

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The subject of this paper is a description of an academic staff evaluation model developed and successfully tested on Palacký University Olomouc - Faculty of Science. We have examined various academic staff evaluation models used on universities both in the Czech Republic and abroad. These models were subjected to a detailed analysis regarding their practical aspects and their mathematical solution. This resulted in the design of several academic staff evaluation models, differing in the ways members of academic staff are evaluated in separate areas of their activity and in the aggregation method for these partial evaluations (weighted average, OWA, WOWA). Considering the faculty management requirements for the evaluation function shape, the following model was finally chosen. General requirements for the model were as follows: It should (1) include preferably every aspect of academic staff activity, (2) use only easily proven and objective data and (3) the work with the model should be as simple as possible. Other requirements were for the final evaluation (4) to maximally reflect staff benefits to the faculty and (5) not to be a simple average of partial evaluations in separate areas of activity. The desired output of the model was not to arrange members of academic staff in order of their performance, nor to obtain a single number interpretable only with difficulty. A

basic piece of information about both focus and performance of the academic staff was considered sufficient. Such assignment implied the use of linguistically oriented fuzzy modeling - linguistic variables, rule bases and approximate reasoning (i.e. of fuzzy expert systems). The performance of each member of academic staff is evaluated in both pedagogical and research and development (R&D) area of activities. Input data are acquired from a form filled-in by the staff where single activities are assigned a score according to their importance and time-consumption. We take three areas into consideration for pedagogical performance evaluation: (a) direct teaching, (b) supervision of students and (c) work associated with the development of the fields of study. The research and development activities evaluation is based on the R&D methodology valid in the Czech Republic but other important activities (grant project management, editorial board memberships etc.) are also included. Both pedagogical and R&D areas are assigned a standard score - different for senior assistant professors, associate professors and professors. The number representing member of staff partial evaluation in certain area is determined as a multiple of the respective standard for his or her position. For a better clarity and easier interpretation are these numbers transformed into a verbal evaluation using linguistic scale values. A linguistically oriented fuzzy expert system is therefore used to aggregate both partial evaluations - for pedagogic and R&D area of activities. The main advantage of this type of aggregation is that it allows to set-up the shape of the aggregation function completely in accordance with evaluator's requirements (e.g. to appreciate excellence achieved in one of the areas). This type of aggregation is transparent and comprehensible even to a layman as it is described in linguistic terms. The overall aggregated evaluation is also available as a linguistic expression. Our model also takes into account academic offices and management activities load for each member of academic staff (understood here as activities draining his or her time capacity to achieve maximum performance in each area). The overall aggregated evaluation for pedagogic and R&D activities is aggregated with the rate this "activity load" criterion is met by means of another fuzzy expert system. This results in a language description of members of academic staff overall work load.

PERPENDICULAR PURSUIT MOTION AS A MODEL OF NONHOLONOMIC MECHANICAL SYSTEM

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Nonholonomic mechanical systems (i.e. Lagrangian systems subjected to constraints on time, positions and velocities) are very popular in many scientific domains in recent years. To solve problems in nonholonomic mechanics we have a choice between two methods: deformed equations (containing Lagrange multipliers) arising from deformation of the original mechanical system by constraint force, or reduced equations, which are derived from a nonholonomic variational principle. The aim of this paper is to show both of these methods on one example of a perpendicular pursuit of an inclined throw in the gravitational field.

MATHEMATICAL ANALYSIS OF A CLASS OF PATH-DEPENDENT OPTIONS

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In our work we investigate Asian options. In the first part, we explore statistical properties of a time integral of the geometric Brownian motion. We approximate this integral by a log-normal distributed random variable. Then, using a suitable copula function, we price the average strike Asian option. In second part, we focus on the early exercise boundary problem for American-style Asian options. We generalize algorithm based on transformation methods to the case of an exponentially weighted arithmetic averaged Asian option and to geometric averaged Asian option.

ANOTHER APPROACH TO MINING OF LINGUISTIC ASSOCIATIONS

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Keywords: Association mining, evaluative linguistic expression, linguistic association, IF-THEN rule

Association mining is a method for searching associations among attributes of a given data set that can be expressed in a form of natural language sentences. The theoretical foundations of this method were published in [1]. Recently we extended the applicability of the original model established in the mentioned paper since we presented a way how to use standard notions of the fuzzy mathematics, namely, the fuzzy partition and fuzzy covering [2], in the association mining process.

Our current aim is to improve linguistic association mining in a way allowing us to work with found linguistic associations in a more effective and intuitive way.

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A DATA ENVELOPMENT ANALYSIS-FUZZY ANALYTIC HIERARCHY PROCESS COMBINED METHODOLOGY TO EVALUATE THE PERFORMANCE OF INTERNATIONAL AIRPORTS IN TURKEY

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Keywords: Fuzzy Analytical Hierarchy Process (FAHP), Triangular Fuzzy Numbers (TFN), Data Envelopment Analysis (DEA), Airport

Data Envelopment Analysis (DEA) is a known method to determine the efficient and inefficient units in concern. There are five international airports in Turkey. This study aims the airport selection and comparison system base on Fuzzy Analytical Hierarchy Process (FAHP). The FAHP is applied to evaluate the best airport with rating criteria. These criteria classify other five airport inputs such as capacity and operational activities and outputs. The result obtained by FAHP is compared with result produced by Mikhailov weighted goals method. The FAHP method adopted here uses Triangular Fuzzy Numbers (TFN). The methodology is demonstrated by use of a performance evaluation problem of the airports. The characteristics of major international airports in Turkey that impact the operations are defined by DEA-AHP.

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CLASSIFICATION OF ASSOCIATION COEFFICIENTS AND ITS APPLICATION

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In research in various fields of human life, the groups/clusters-forming in set of all objects allows clearer view and better orientation among the objects. The concepts of similarity and dissimilarity play a key role in this area. Cluster analysis addresses this very issue: the objects inside the cluster have the greatest degree of similarity while the clusters themselves show the highest rate of dissimilarity. The study of cluster analysis brings the question of correctness of the constructed clusters.

This problem can be analyzed differently according to the narrow context of the interest in studied area, e.g. the social, physical, educational and other fields of research. This paper uses the hydrological data which are processed by two different methods of clustering. The aim of this work is to classify the most commonly used association coefficients in engineering practice and to apply them to assess the results of different methods of clustering using the principle of similarity.

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PERCEPTION-BASED LOGICAL DEDUCTION AND THE PROBLEM OF REDUNDANT FUZZY RULES

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The *perception-based logical deduction* [2] is a method of deducing conclusions on the basis of a *linguistic description*, i.e., a finite set of fuzzy IF-THEN rules:

$$\text{IF } X \text{ is } \mathcal{A}_1 \text{ THEN } Y \text{ is } \mathcal{B}_1$$

$$\dots\dots\dots\dots\dots\dots$$

$$\text{IF } X \text{ is } \mathcal{A}_m \text{ THEN } Y \text{ is } \mathcal{B}_m$$

where $\mathcal{A}_1, \dots, \mathcal{A}_m, \mathcal{B}_1, \dots, \mathcal{B}_m$ are so called *evaluative linguistic expressions* (e.g. small, very small, roughly medium, big).

Some practical problems, such as time series analysis [1] require to generate a linguistic description automatically from a given data. But such linguistic descriptions usually suffer from several problems. The existence of redundant fuzzy rules is among them.

As it has been pointed out in [3] that an intuitively redundant rule (antecedent fully overlapped by antecedent of another rule with an identical consequent) does not have to be necessarily redundant with respect to a formal definition of the

redundancy. Therefore, a further investigation of this problem is highly desirable. This presentation is a contribution to this topic.

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ASSESSMENT OF UNIVERSITY DEPARTMENTS BASED ON MULTICRITERIA DECISION MAKING AND AGGREGATION OPERATORS

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Keywords: alternative, preference structure, multicriteria method, aggregation operator

In order to solve decision making problem we have to compare, evaluate, rank and choose the best alternative x from the set of alternatives X. The ranking of alternatives is represented by a preference structure. In this paper we want to show more than one approach how to create the preference structure of alternatives. These approaches lead us to use chosen multicriteria decision methods and aggregation operators.

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KERNEL CHOOSING WITH RESPECT TO THE BANDWIDTH IN KERNEL DENSITY ESTIMATES

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Keywords: kernel, bounded support, density estimation

Kernel density estimates belong to the most popular nonparametric density estimates.

It is a known fact, that these estimates depend on a bandwidth, which controls the smoothness of

the estimate, and on a kernel, which plays a role of a weight function.

E.g. in [1, 2] were presented methods for finding the

optimal bandwidth.

We focus on the kernel function choice, especially on kernels with a bounded support.

In [3] is suggested a criterion for the optimal kernel. Our aim is to study

the optimality of the kernel with respect to the bandwidth choice.

A simulation study brings comparison of the kernels. It shows up, that the cosine kernel performs better

than the frequently used Epanechnikov kernel.

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A NOTE ON LARGE CAYLEY GRAPHS OF DIAMETER TWO AND GIVEN DEGREE

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Let $C(d, 2)$, $AC(d, 2)$, and $CC(d, 2)$ be the largest order of a Cayley graph of a group, an Abelian group, and a cyclic group, respectively, of diameter 2 and degree d. The currently known best lower bounds on these parameters are $C(d, 2) \geq (d+1)^2/2$ for degrees $d = 2q-1$ where q is an odd prime power, $AC(d, 2) \geq (3/8)(d^2-4)$ where $d = 4q - 2$ for an odd prime power q, and $CC(d, 2) \geq (9/25)(d + 3)(d - 2)$ for $d = 5p - 3$ where p is an odd prime such that $p \equiv 2 \mod 3$.

For diameter two we present a construction for 'large' Cayley graphs of semidirect products of groups out of 'small' Cayley graphs of cyclic groups, such that the ratio of the order of the graph to the square of the degree of the graph is approximately the same for both the input and the output graphs. As a consequence we obtain a lower bound on $C(d, 2)$ of the form $(9/25)d^2 - O(d)$ for a much larger variety of degrees than those listed above.

FUZZY SETS IN HR MANAGEMENT

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The aim of this paper is to demonstrate the possible applications of fuzzy sets in the human resources management. At first, the using of the employess' evaluation will be demonstrated. The evaluation of employees is based on multiple criteria evaluations. The criteria are derived by using the typical competencies of the employee. For any given role, a competency model has been created with different normalized weights assigned to various competencies. The evaluation process proceeds in the following way: at first, the appointed evaluators fill in questionnaires indicating to what extend, in their view, the tested employee meets his/her competencies. These evaluations are expressed by using the linguistic fuzzy scales. The level of competencies fulfilment of the employee is calculated as weighted average of fuzzy evaluations conducted by each of the evaluator. Then, the overall fulfilment level of the employee's role, again as a weighted average of fuzzy numbers, is calculated according to the specified model.

Next possibility is based on the competency models and multiple criteria evaluation too. The main difference is in the purpose - in this case we want to specify the suitability of the given worker for the given role. For any given employee, the evaluation described in the previous part will be calculated. In the end, the overall fulfilment level as a weighted average of fuzzy numbers is calculated according to the all competency models. The best evaluation identified the most suitable role for the given employee.

Finally, the last application presented in this paper is in area of working team design. On basis of Belbin's team role theory, the composition of team roles for the

given employee is represented by normalized fuzzy weights. For the given working role the required composition of team roles will be created. This composition presents uncertain unit's distribution and is expressed by fuzzy weights. In the following step the composition of team roles for the given employee will be created. In the both levels the Belbin's questionnaire will be used. Finally, the comparision of employees' team roles and team roles of the given working role will be done.

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