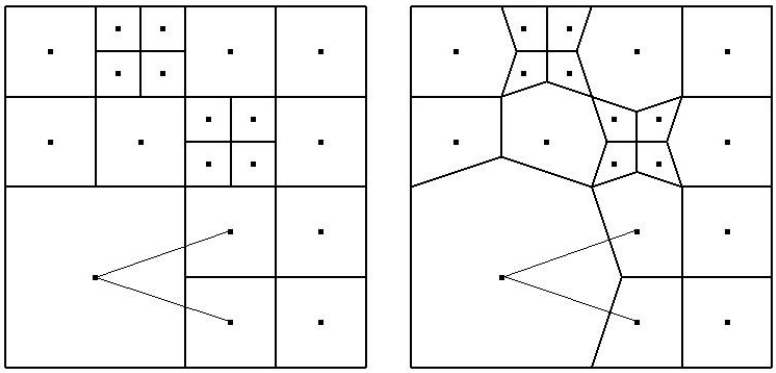
Adaptívna metóda konečných objemov na riešenie lineárnej difúznej rovnice na konzistentnej adaptívnej mriežke
Adaptive Adaptive Finite Volume Method to solve the Linear Diffusion Equation on a Consistent Quadtree Grid

Zuzana Krivá, Karol Mikula

In this paper we present a novel algorithm to solve the linear diffusion equation on the nonuniform quadtree grid adapted for the finite volume method used for the space discretization. This grid, depending on the data, is built using the quadtree technique and is procedurally modified in such way, that the connection of representative points of two adjacent finite volumes is perpendicular to their common boundary. This property is important when we use the finite volume discretization. Examples of basic quadtree and consistent grids are displayed in the fig. 1.



Obr. 1. ***An example of the original quadtree grid together with the representative points of its elements (on the left). This grid is transformed into the consistent one (on the right).***

The paper presents the implicit finite volume numerical scheme, its EOC and shows some outputs of the adaptive algorithm for selected noisy data.

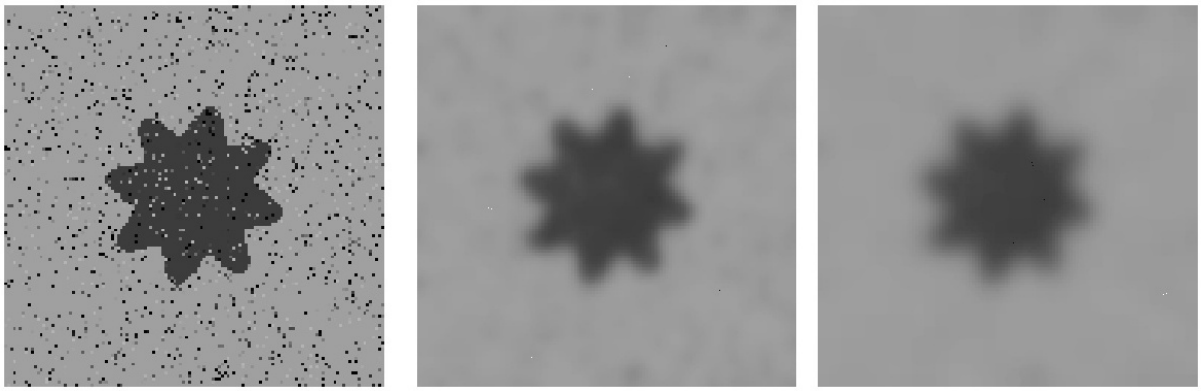
Tabuľka 1. ***EOC calculations, from the left:*** h ***means the size of finite volumes in the initial grid if there would be no coarsening,*** τ ***is the time step,*** N ***is the number of time steps performed over*** $T = [0.5, 0.6]$***. Then the errors*** $E(h)$ ***and*** EOC ***for both Grid a) and Grid b) are given.***

h	τ	N	Grid a) $E(h)$	Grid a) EOC	Grid b $E(h)$	Grid b) EOC
$\frac{1}{16}$	0.003906	25	0.0188		0.01302	
$\frac{1}{32}$	0.000977	102	0.00465	2.015	0.003459	1.912
$\frac{1}{64}$	0.000244	409	0.001154	2.010	0.00089	1.958
$\frac{1}{128}$	0.000061	1638	0.000288	2.002	0.000109	1.996

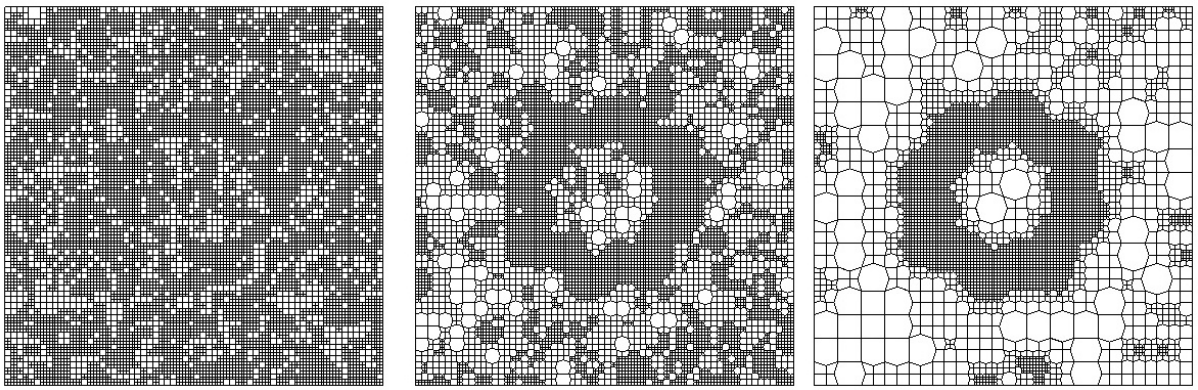
The presented adaptive numerical scheme can become the stepping stone to solve the nonlinear diffusion equations on this type of grid.

Experiment 1. This experiment was performed on the data of the size $128 \times 128 = 16384$ elements spoilt by 10% salt&pepper noise. The initial grid contained 12658 elements, after 5 scale

steps 9835 elements and after 10 scale steps only 4615. The decrease of elements was caused by different type of noise and also by the fact, that the original image covered less part of the total image area than in the previous experiment. The smoothed data and corresponding grids are shown in the fig.2 and the fig.3.



Obr. 2. ***On the left: the original noisy data. In the middle: the 4th scale step. On the right: the 20th scale step.***



Obr. 3. ***On the left: the initial grid (12 658 elements). In the middle: the grid in the 4th scale step (9835). On the right: the final grid (the*** 10^{th} ***time step with 4615 elements).***

Key words: Image processing, the linear heat equation, finite volume method, adaptivity.
Kľúčové slová: Spracovanie obrazu, lineárna rovnica vedenia tepla, metóda konečných objemov, adaptivita.

Acknowledgements.
The work on this paper has been supported by the grant APVV-0184-10.