Identification of the capillary conduction coefficient from experimental data
Identifikácia súčiniteľa kapilárnej vodivosti z experimentálnych údajov

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Identification of capillary transfer properties, namely in the case of porous building materials, is a serious nontrivial problem, both from the experimental and the computational point of view. An increasing moisture content in materials and structures can make their thermal insulation and accumulation properties much worse, which forces unexpected high energy consumption, and contribute to the deterioration of their mechanical properties substantially, too. Moreover, some technologies, as in cooling and freezing plants, should not admit any presence of humidity from external environment at all. Reliable computational modelling and simulation, supported by well-considered organization of experiments, is thus very desirable.

Unlike the linearized theory of heat conduction, elastic deformation, etc., even in the case of a (seemingly easy) direct problem of capillary conduction (with a priori known material properties), the nonlinear and nonstationary character of the analyzed physical process brings non-negligible difficulties to all computational tools. Most identification approaches try to overcome such obstacles, as well as still other difficulties, typical for inverse problems, namely ill-possedness, numerical instability, etc. – cf. [Isa], p. 20, using very simple model configurations of experiments. However, simplified considerations related to one-dimensional specimens of infinite lengths and other phenomena, not observable in the real world, but forcing numerical integration over infinite sets from small data sets with hidden (both system and random) errors from various sources, do often not lead to satisfactory results. This is a principle motivation for our checkup of existing approaches and for some recommendations to their improvement with the aim of derivation of effective computational algorithms producing credible results for building design.

Considering an open set Ω in the 3-dimensional real Euclidean space, supplied with the Cartesian coordinate system, and some time range $I = (0, \dots)$, we are able to write the mass conservation principle for unknown mass fraction $u(x,t)$ with $x \in \Omega$ and $t \in I$ as

$$(v, \dot{u}) = \langle \nabla v \cdot \nu, \bar{j} \rangle + (\nabla v, j) \tag{1}$$

for any test function from an appropriate function space; $(.,.)$ here denote the integrals of function products over Ω, $\langle .,. \rangle$ the similar ones over its boundary $\partial\Omega$, $j = \bar{j}$ is prescribed on $\Gamma \subseteq \partial\Omega$, as well as $u = \bar{u}$ on $\Theta = \partial\Omega \setminus \Gamma$ and $u(.,0) = u_0$ to set the initial status, moreover dots mean partial derivatives with respect to t and ν local vectors of outward unit normals. Moreover, the empirical Fick constitutive relation

$$j = -\nabla \beta(u) \tag{2}$$

is available. The crucial problem of identification of the capillary conduction property is to identify the coefficient $\kappa(u) = \beta'(u)$ thanks to some additional information about the distribution of $u(x,t)$, e. g. using the microwave technique.

Nearly all identification approaches, generating reasonable computational algorithms, can be derived from (1) and (2), whose most useful common variational formulation is

$$(v, \dot{u}) + (\nabla v, \kappa(u) \nabla u) = \langle v, \bar{j} \rangle \,; \tag{3}$$

alternative formulations (including the classical differential ones) to (3) can be derived using the Green - Ostrogradskiĭ theorem together with some facts from the theory of distributions and with the properties of (abstract) functions from Lebesgue, Sobolev and Bochner spaces. The rough list of these approaches (in more details discussed in the full paper, prepared for *Forum Statisticum Slovacum*) contain:

a) the general variational method for special test functions with respect to known data,

b) the first integration method, exploiting the Dirac measures and their relations to Laplace operators,

c) the second integration methods, using the derivatives $\kappa'(u)$ and converting the problem to certain nonlinear ordinary differential equation,

d) the least squares method, namely for the post-processing to b), c) and most algorithms involved in the group e),

e) the extensive group of methods related to special geometrical configurations.

Inside e) it is useful to distinguish:

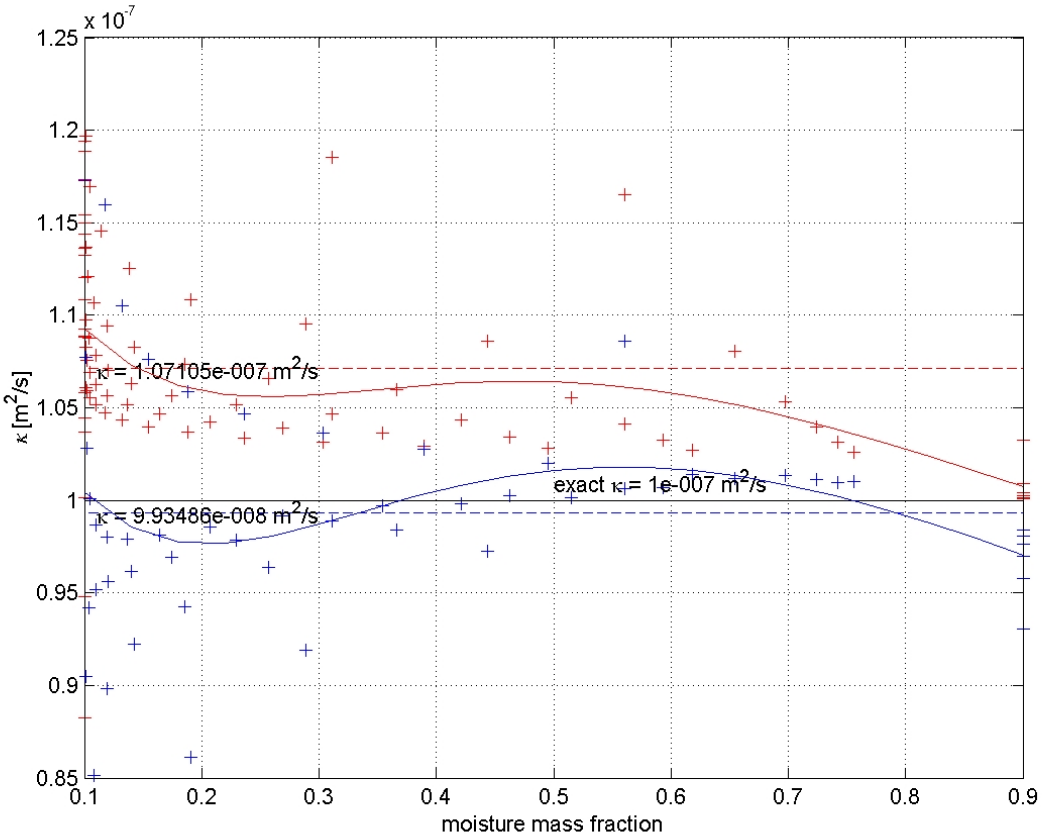
e1) the classical Boltzmann - Matano method, i. e. the application of the Boltzmann transform (1894) by Matano (1933) to a special one-dimensional problem,

e2) the revised Matano method, removing the Boltzmann transform and generalizing some assumptions,

e3) the modification of e2) without integrals over infinite sets (to improve the precision of numerical computations),

e4) the double integration method, relying on the analysis of parametric equations of isolines for functions $u(x,t)$, avoiding d).

All utilized measurement data come from the laboratory of the Faculty of Civil Engineering of the Brno University of Technology (BUT). The original identification software has been written in MATLAB, with no references to additional packages. The illustrative figure compares the identification results from e1) (the upper curve) and e3) (the lower curve), both combined with d), using the discrete basis of exponential functions. The nearly constant values κ were expected; thus also the result, taken from the analytic expression, assuming constant κ including the Gauss error function, is presented.



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