# On the representation of mixtures in the ESR model for QM

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It is well known that the attempt at solving the *objectification problem* [1, 2, 3] of the quantum theory of measurement did not lead to largely shared solutions. The problem occurs because the properties of a physical system may be *nonobjective* according to the standard interpretation of quantum mechanics (QM), which is a consequence of the *contextuality* and *nonlocality* of QM. The Lecce group on the Foundations of QM has proposed some years ago a *Semantic Realism* (SR) interpretation of QM [4, 5] which restores objectivity on the basis of a criticism to known no-go theorems [6, 7, 8]). Successively the group has worked out an *Extended Semantic Realism* (ESR) model [9, 10, 11, 12, 13, 14] that modifies the SR interpretation and provides an extension and reinterpretation of QM which avoids the objectification problem and some ensuing paradoxes. Basically, the ESR model is a noncontextual, hence local, hidden variables (*h.v.*) theory which circumvents the no–go theorems by modifying the interpretation of quantum probabilities. To this end *microscopic properties* of individual examples of physical systems (or *physical objects*) are introduced as theoretical entities, together with *generalized macroscopic observables* that represent instead the observational entities of the model. These observables are obtained from the standard observables of QM by adding a *no-registration outcome* to the set of possible values of each observable of QM and making the crucial assumption that one can get such an outcome because of the microscopic properties (the h.v.) possessed by the physical object also in the case of idealized measurements with efficiency equal to 1. It follows that the correspondence between theoretical microscopic properties and observational macroscopic properties (defined as pairs $(A_0, X)$, with $A_0$ a generalized observable and $X$ a Borel set on the real line) is not one–to–one. This feature of the ESR model suggests some assumptions at the observational level which allow one to recover the quantum rules for calculating probabilities in the case of pure states, yet reinterpreting such probabilities as conditional on detection rather than absolute. Therefore a broader formalism must be introduced in the ESR model to calculate absolute probabilities, and the predictions of the ESR model do not coincide with the predictions of QM, not even when they are formally identical. But the difference between the predictions of the two theories depend on some parameters (the *detection probabilities*) which may be so small that it remains unnoticed in many cases. There are however physical situations in which it becomes relevant and can be used to disprove or confirm the ESR model [12, 14].

Even if the ESR model must be taken as a whole, the macroscopic part of it can be presented independently, as a new theory which modifies the interpretation of the basic Hilbert space formalism of QM and embodies it into a broader mathematical framework. One can then try to extend the new theory to mixtures. To this end it is important to remind that various authors maintain that there are in QM two classes of mixtures, that is, *proper* and *improper* mixtures [1, 15, 16], while other authors argue that only improper mixtures exist in QM [17, 18, 19, 20]. From a mathematical point of view no distinction occurs because all mixtures are represented by density operators, but there are ambiguities and problems in the interpretation of the coefficients of the various possible expressions of the density operators [19]. One can, however, point out some fundamental differences between the physical definitions of the two kinds of mixtures. Because of these differences the mathematical representations of proper and improper mixtures do not coincide in the ESR model. To be precise, one can specify two classes of *operational definitions*, one referring to proper and the other to improper mixtures. By using the operational definitions of proper mixtures one can show that each proper mixture is represented in the ESR model by a familiy of density operators rather than by a single density operator, and that different operational definitions correspond to different mathematical representations [14, 21]. By using the operational definitions of improper mixtures one can show that the mathematical representation of an improper mixture in the ESR model coincides with the representation provided by MQ. Hence proper and improper mixtures are neatly distinguished in the ESR model, and the probabilistic predictions of this model neither formally coincide with the predictions of QM in the case of proper mixtures. This offers a solution of the problems raised by the standard quantum representation of mixtures and allows one to propose a scheme for an experiment aiming to check whether the predictions of QM or the predictions of the ESR model are correct.

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