Atoms in Archimedean lattice effect algebras and its sub-lattice effect algebras

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Abstract

Lattice effect algebras are common generalizations of MV-algebras and orthomodular lattices ([2], [3], [6]). Thus they may include noncompatible pairs of elements as well as elements which are unsharp, fuzzy or imprecise. It was shown that the set $S(E)$ of all sharp elements of a lattice effect algebra E is an orthomodular lattice, [5]. Further, the set $B(E)$ of all those elements which are compatible with every element of E is an MV-algebra (MV-effect algebra). Thus the set $C(E)$ of all elements which are sharp and simultaneously compatible with all elements of the lattice effect algebra E is a Boolean algebra which is the intersection of $S(E)$

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and $B(E)$ ([4], [8], [9]). Finally, every lattice effect algebra E is a set-theoretical union of MV-algebras called blocks of E. In fact a block of E is a maximal subset of pairwise compatible elements of E, [10].

Every atom (a minimal nonzero element) of a block of E is also an atom of E. On the other hand an atomic effect algebra E (i.e., with an atom under every nonzero element) may have non-atomic block (see [1]). Moreover if $S(E)$ or $C(E)$ are atomic then E may be atomless, e.g., for standard effect algebra of real numbers in the interval $[0, 1]$.

We present some families of Archimedean atomic lattice effect algebras in which $S(E)$ or $C(E)$, resp. $B(E)$ or every block of E is atomic for every E of these families.

Some applications of mentioned results in problems on the existence of states, resp. the smearing of states from sharp elements onto E can be shown, [11].

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