Archimedean atomic lattice effect algebras with almost orthogonal set of atoms

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Abstract

Archimedean atomic lattice effect algebras present an important family of lattice effect algebras ([1], [2], [3]), which are common generalizations of orthomodular lattices and MV-algebras. Properties of their sets of atoms (i.e., minimal nonzero elements) are in several cases substantial for algebraic structure or topological properties of atomic lattice effect algebras. For instance the "Isomorphism theorem based on atoms" for Archimedean atomic lattice effect algebras can be proved [4].

We show that a set of all atoms in an Archimedean atomic lattice effect algebra E is almost orthogonal iff E is an order continuous lattice and its the interval topology

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is Hausdorff. Here the almost orthogonality means that the set of non-orthogonal atoms to any atom of E is finite (for orthomodular lattices see e.g. [5]).

Further, in the case when E has only finitely many blocks (i.e., maximal subsets of pairwise compatible elements) the almost orthogonality is equivalent with order continuity of lattice operations in E. Moreover, for those so called block-finite Archimedean atomic lattice effect algebras the almost orthogonality of E is equivalent with the almost orthogonality of the Mac-Neille completion $MC(E)$ of E which is equivalent with the condition that $MC(E)$ is compactly generated.

References

[1] A. Dvurečenskij, S. Pulmannová: New Trends in Quantum Structures, Kluwer Acad. Publ., Dordrecht/Ister Science, Bratislava 2000.

[2] D.J. Foulis, M.K. Bennett, Effect algebras and unsharp quantum logics, Found. Phys. 24 (1994), 1325–1346.

[3] F. Kôpka, F. Chovanec, D-Posets, Math. Slovaca 44 (1994), 21–34.

[4] J. Paseka, Z. Riečanová, Isomorphism theorems on generalized effect algebras based on atoms, Information Sciences (2009), 179 521–528.

[5] S. Pulmannová, Z. Riečanová,Block-finite atomic orthomodular lattices, Journal Pure and Applied Algebra 89 (1993), 295–304.