Simplex-valued states

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Abstract

In [7], [4], [5], [8], [9], we studied a model of probability based on D-posets of fuzzy sets (cf. [2], [3]). The idea is to start with the domain $I = [0, 1]$ of states (probability measures) and to cogenerate the category ID of D-posets of fuzzy sets in which observables and states are morphisms. At the QS'08 conference in Sopot, replacing I by a suitable cogenerator C and the category ID by a suitable cogenerated category, we have generalized the model one step further ([6]). In particular, motivated by IF-probability theory (intuitionistic fuzzy, cf. [1], [10]), we proposed to study n-component probability domains in which each event represents a body of competing components and the range of a state represents a simplex S_n of n-tuples of possible "rewards"—the sum of the rewards is a number from $[0, 1]$. For $n = 1$ we get fuzzy events, for example a bold algebra, and the corresponding fuzzy probability theory can be developed within the category ID of D-posets (equivalently effect algebras) of fuzzy sets and sequentially continuous D-homomorphisms. For $n = 2$ we get IF-events, i.e., pairs (μ, ν) of fuzzy sets $\mu, \nu \in [0, 1]^X$ such that $\mu(x) + \nu(x) \leq 1$ for all $x \in X$, but we order our pairs (events) coordinatewise. Hence the structure of IF-events (where $(\mu_1, \nu_1) \leq (\mu_2, \nu_2)$ whenever $\mu_1 \leq \mu_2$ and $\nu_2 \leq \nu_1$) is different and, consequently, the resulting IF-probability theory models a different principle. In this talk we present new results dealing with S_n-valued states.

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