The State Context Property System of a contextual but non-quantum model

Bart D'Hooghe*

In this paper we adopt an operational approach to quantum mechanics in which a physical entity is determined by the structure of its State Context Property System (SCoP) [1, 2, 3, 4, 5]. A state context property system $(\Sigma, \mathcal{M}, \mathcal{L}, \mu, \xi)$, consists of three sets Σ, $\mathcal{M}$, $\mathcal{L}$ and two functions μ and ξ, such that

$$\mu \quad : \quad \mathcal{M} \times \Sigma \times \mathcal{M} \times \Sigma \rightarrow [0,1] \tag{1}$$

$$\xi \quad : \quad \Sigma \rightarrow \mathcal{P}(\mathcal{L}) \tag{2}$$

The sets Σ, $\mathcal{M}$ and $\mathcal{L}$, play the role of the set of states Σ, the set of contexts $\mathcal{M}$, and the set of properties $\mathcal{L}$ of an entity S. The function μ describes transition probabilities between couples (e, p) and (f, q) of contexts $e, f \in \mathcal{M}$ and states $p, q \in \Sigma$, while the function ξ describes the sets of actual properties $a \in \mathcal{L}$ for the entity S being in different states $p \in \Sigma$.

To illustrate this approach, we consider a model for a spin-1/2 entity in which the maximal change of state of the system due to interaction with the measurement context is controlled by a parameter which corresponds with the number N of possible outcomes in an experiment. In the limit $N = 2$ the system reduces to a model for the spin measurements on a quantum spin-1/2 particle [6]. In the other limit $N \rightarrow \infty$ the system is classical, i.e. the experiments are deterministic and its set of properties is a Boolean lattice [7]. For intermediate values of N two of the axioms used in Piron's representation theorem are violated [8, 9], namely the covering law and weak modularity. For a modified version of this model it is even impossible to define an orthocomplementation on the set of properties [10]. Another interesting feature in the intermediate situations of this model is that the probability of a state transition in general not only depends on the angular distance between the two states but also on the measurement context which induces the state transition. This justifies the use of SCoP to represent such system. This suggests that transition probability should not be regarded as a secondary concept which can be derived from the structure on the set of states and properties (via Gleason's theorem), but instead should be regarded as a primitive concept by its own right for which the measurement context is crucial, and that only under very special 'quantum conditions' it can be reduced to a secondary concept.

Acknowledgments This work was carried out within the project G.0234.08 of the Research Programme of the Research Foundation–Flanders (FWO).

*Leo Apostel Center for Interdisciplinary Studies, Vrije Universiteit Brussel (VUB), Belgium; e-mail: bdhooghe@vub.ac.be

References

[1] Aerts, D. (2002). Being and change: foundations of a realistic operational formalism. In D. Aerts, M. Czachor and T. Durt (Eds.), *Probing the Structure of Quantum Mechanics: Nonlinearity, Nonlocality, Probability and Axiomatics*. Singapore: World Scientific, 71–110. Archive reference and link: http://uk.arxiv.org/abs/quant-ph/0205164.

[2] Aerts, D. and Gabora, L. (2005). A theory of concepts and their combinations I: The structure of the sets of contexts and properties. *Kybernetes* **34**, 167–191. Archive reference and link: http://uk.arxiv.org/abs/quant-ph/0402207.

[3] Aerts, D. and Gabora, L. (2005). A theory of concepts and their combinations II: A Hilbert space representation. *Kybernetes* **34**, 192–221. Archive reference and link: http://uk.arxiv.org/abs/quant-ph/0402205.

[4] Flender, C. (2008). Ecological Modelling of Information Systems. *Proceedings of the Doctoral Consortium at the 20th International Conference on Advanced Information Systems Engineering (CAiSE 2008)*, 17 June 2008, Montpellier, France.

[5] Flender, C., Kitto, K. and Bruza, P. (2009). Beyond Ontology in Information Systems. *Proceedings of the Third Quantum Interaction Symposium (QI 2009)*, 25–27 March 2009, *Lecture Notes in Artificial Intelligence*, vol **5494**, Springer, 2009.

[6] Aerts, D. (1986), A possible explanation for the probabilities of quantum mechanics. *Journal of Mathematical Physics* **27**, 202–210.

[7] Durt, T. and D'Hooghe, B. (2002), The classical limit of the lattice-theoretical orthocomplementation in the framework of the hidden-measurement approach. In D. Aerts, M. Czachor and T. Durt (Eds.), *Probing the Structure of Quantum Mechanics, Nonlinearity, Nonlocality, Computation and Axiomatics*. Singapore: World Scientific, 111–129.

[8] Piron, C. (1976), *Foundations of quantum physics*. Reading, Mass.: W. A. Benjamin.

[9] Aerts, D. (1981), *The one and the many: towards a unification of the quantum and the classical description of one and many physical entities*. Doctoral dissertation, Vrije Universiteit Brussel (VUB).

[10] D'Hooghe, B. (submitted). On the orthocomplementation of State–Property–Systems of contextual systems. To appear in *International Journal of Theoretical Physics*.