Bell's inequalities and Pauli matrices

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Bell's inequalities play an important role in experimental tests of physical theories. These inequalities provide a bound on a correlation experiment between two systems which are no longer interacting but have interacted in the past.

We generalize Bell's inequalities (Cirel'son's type) to all complex linear spaces and general correlation dualities given by semidefinite sesquilinear forms. More precisely, let Q be the semidefinite sesquilinear form defined on the linear complex space X then

$$\frac{1}{2}\left|Q\left(a_1, b_1+b_2\right)-Q\left(a_2, b_1-b_2\right)\right| \leq \sqrt{2}, \tag{1}$$

for all $a_1, a_2, b_1, b_2 \in X$ whose seminorm (given by Q) is less or equal to one. Our arguments in the proof are based only an appropriate application of the Schwartz inequality and do not require advanced operator algebraic techniques (compare with [1, 2]).

Suppose further that $\mathcal{A}$ is a complex *-algebra and φ is a faithful state on $\mathcal{A}$. Then φ induces naturally a positive sesquilinear form Q on $\mathcal{A}$ by $Q(x, y) = \varphi(y^* x)$. Our main result shows that the bound $\sqrt{2}$ in (1) is attained if and only if the elements a_1 and a_2 are the realization of the Pauli spin matrices. Moreover, in important special cases φ restricts to the trace on the algebra generated by a_1 and a_2. Consequences of our results for the structure of Bell's inequalities and entanglement are discussed.

References

[1] S. J. Summers, R. Werner: *Bell's Inequalities and Quantum Field Theory, I: General Setting*, J. Math. Phys., **28** (1987), 2448-2456.

[2] S. J. Summers, R. Werner: *On Bell's Inequalities and Algebraic Invariants*, Lett. Math. Phys., **33** (1995), 321-334.