A New Representation of Generalized Observables for a Hilbert Space Extension of QM

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Most quantum physicists maintain that contextuality and nonlocality are logical consequences of the mathematical apparatus of quantum mechanics (QM) that cannot be avoided if one does not want to reject QM together with its outstanding success in providing explanations and predictions of physical facts. This belief is strongly based on known theorems [1]–[3], and the recent developments in quantum information assume contextuality and nonlocality as standard interpretational tools when dealing with entangled states and related issues. The research group of Lecce on the foundations of QM, however, has shown many years ago that the proofs of the aforesaid theorems depend on an implicit epistemological assumption (*metatheoretical classical principle*, or *MCP*) which can be disputed from a quantum viewpoint. Whenever MCP is replaced by a weaker principle (*metatheoretical generalized principle*, or *MGP*) such proofs cannot be completed, which implies that noncontextual and local interpretations of the mathematical formalism of QM are not *a priori* excluded. The research group of Lecce has therefore worked for a long time to provide an example of interpretation of this kind (*semantic realism*, or *SR*, interpretation [4]–[6]) and, successively, to supply set–theoretical models showing that the SR interpretation is consistent. The last model of this kind (*extended semantic realism*, or *ESR*, model) has been recently developed in such a way that it can be now considered as an independent theoretical proposal, according to which the mathematical formalism of QM is embodied into a broader noncontextual and local framework and the measurement problem disappears [7]–[12]. This proposal can be considered as a new kind of hidden variables theory for QM that reinterprets quantum probabilities as conditional instead of absolute. Because of such a reinterpretation MGP holds in the ESR model rather than MCP and, more important, the model yields some predictions that are formally identical to those of QM but have a different physical interpretation and further predictions that differ also formally from those of QM [9], [11]–[13]. Thus, at least in principle, the ESR model can be empirically checked. However, it is still incomplete because the generalized observables introduced in it have only a partial mathematical representation and no general theory of the detection probabilities that appear in it is provided. We intend to fill here the first of these gaps and complete some previous work on this topic [14, 15] by supplying a general mathematical representation for generalized observables which holds if a plausible physical assumption is fulfilled. This representation closely reminds the representation of unsharp observables in unsharp QM [16, 17] but differs from it because of two important features. From one side it is more general, because any generalized observable is represented by a *family* of positive operator valued (POV) measures (parametrized by the unit vectors representing pure states) rather than by a *single* POV measure. On the other side it is less general, because only commutative POV measures occur in the representation of a generalized observable (but it must be stressed that only *idealized* measurement procedures are considered in the ESR model which correspond to sharp measurements in unsharp QM). In addition, the new representation suggests a straightforward generalization of the *projection postulate* of standard elementary QM, which does not imply however any "actualization" of "potential" physical properties because all properties are "actual" in the ESR model, even if they may be unknown or unknowable in specific physical situations. Finally, the generalization of the projection postulate can be justified in the case of discrete generalized observables by introducing a reasonable physical assumption on the evolution of the compound system made up of the (microscopic) physical object plus the (macroscopic) measuring apparatus.

Summing up, we think that the above results are interesting from at least two points of view. Firstly, they provide a concrete example of a theoretical framework that embodies the standard formalism of QM without entailing contextuality and nonlocality, which shows that theories of this kind are abstractly possible even if one does not want to accept the ESR model as a description of natural processes. Secondly, they imply predictions that differ from the predictions of QM, hence the ESR model is falsifiable, even if it may be difficult to contrive experiments able to distinguish it from standard QM because concrete measurements obviously are not "idealized" as the measurements considered by the ESR model.

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