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NUMERICAL MODELLING OF THE GROUNDWATER FLOW IN THE LEFT FLOODPLAIN AREA OF THE DANUBE RIVER *

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Abstract. Interaction of surface and groundwater in left hand-side floodplain area of the Danube River, surrounded by upper power canal and the Danube River was assessed. Two scenarios has been considered for computational analysis. The first scenario is the current state, where dominant influence on the groundwater regime is given by discharge and water level regime in the branch system. Branch system is fed by inlet structure in Dobrohošt'. The second scenario provided in the contribution is a forecast of groundwater regime after a possible construction of planned underwater weirs in the Danube River. Results of numerical modelling solving steady nonlinear Boussinesq equation by the finite element method show that the groundwater regime in the floodplain area would be improved and the drainage effect of the Danube River reduced or completely neglected.

Key words. groundwater flow, finite element method, hydrologic regime, Danube river branch system, floodplain.

1. Introduction. Hydrologic regime of the Danube River before the damming influenced directly the hydrologic regime in river branches as well as the groundwater regime in the floodplain area. In pre-dam conditions the increase of water level in the Danube River bed (see Fig. 1.1, solid line) caused a synchronous increase of water level in the branch system. At discharges higher than 4000 $m^3.s^{-1}$ in the river bed it came to a direct fulfilling of the branch system with water and the branches created together with the Danube a complicated hydraulic surface water system. The studied area of the floodplain region is approximately 62 km^2 where 4000 hectares of productive floodplain forest exists. Most important for surviving of floodplain forest and for improvement of hydrologic conditions in this region is a favourable groundwater regime. The only data concerning the measured groundwater levels before damming were available from the time period 1987-1992. From these data is it apparent that during the vegetation period in pre-dam conditions (before 1992) the groundwater table was in cover layer representing the root zone of the floodplain forest and never decreased into gravel layer, from where no capillary rise is available [3]. The second very important fact was that the fluctuation of groundwater table reached the value 2.5-3.0 m. The lowest groundwater levels were in winter and the highest were in summer according to the fluctuations of water levels in the Danube River. After damming the Danube River the discharges became artificial and due to the power canal scheme of Gabčíkovo waterworks they became lower, as well. The consequence of this hydrologic situation was the decrease of groundwater level in the adjacent area of the Danube. To improve and to control the groundwater regime in the floodplain area an inlet structure situated on the upstream power canal was constructed on Slovak territory in Dobrohošt' (see Fig. 1.1). This structure allows an artificial water supply

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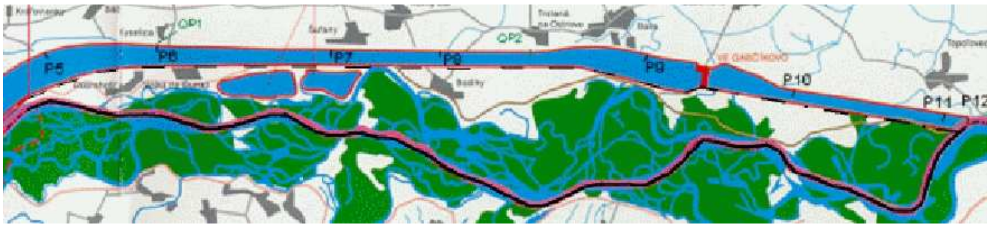

FIG. 1.1. *Map of the area of interest - left hand-side floodplain area.*

into the branch system and consequently into the groundwater. It is dimensioned up to discharge 234 $m^3.s^{-1}$. This quantity would serve not only for water supply into the branch system and for groundwater control but for securing of regulated floods in the floodplain region, as well. The Danube river branch system is an area between flood - protecting dikes and the Danube river. Branch system is a system of cascade hydraulic structures, constructed with culverts including water level regulating enclosure sets and water passes. The closure of culverts for a particular time period can increase the water level in the compartment behind the cascade hydraulic lines, and thus simulate a flood wave or water level fluctuation. Seepage canals (the left seepage canal is indicated by dashed line in Fig. 1.1) collect seepage water on both sides of the reservoir to control increase of groundwater level, and to ensure the groundwater level fluctuations [6]. Downstream power canal (see Fig. 1.1, dash-dot line) takes water from hydropower station Gabčíkovo and navigation locks into the Danube River. Our region of interest (computational domain for the finite element analysis) is bounded by the Danube river bed, left seepage canal and downstream power canal. This region includes Danube river branch system and thus, it is complicated from the point of view of geometrical modelling. We used the finite element method (FEM) for modelling of groundwater regime in this area using ANSYS software [1]. In section 2 a mathematical model describing groundwater flow is presented. In section 3 a computational solution of the problem is described as well as in section 4 results of the numerical solution are discussed.

2. Mathematical model. The groundwater flow in continuous porous media [2] is modelled by the continuity equation

$$\frac{\partial \theta}{\partial t} + \operatorname{div} \bar{q} + w = 0 \tag{2.1}$$

where θ is the water content, $\bar{q}$ is the vector of filtration velocity and w is an intensity abstraction per unit volume. The relation (2.1) represents the law of conservation of mass. Let us express the hydraulic potential ϕ by

$$\phi = \frac{p}{\gamma} + z \tag{2.2}$$

where p is hydrostatic pressure, γ specific weight and z elevation head. Then the empirical Darcy's law says

$$\bar{q} = -k \nabla \phi \tag{2.3}$$

where $\nabla \phi$ is a gradient of the potential and k is a tensor of hydraulic conductivity. Combining the equations (2.1) and (2.3) we get the basic equation of 3D-flow in porous media

$$\frac{\partial \theta}{\partial t} - \text{div } (k \nabla \phi) + w = 0\,. \tag{2.4}$$

In case of homogeneous and isotropic porous media k reduces to a scalar expressing the hydraulic properties of the skeleton. The basic equation (2.4) holds in a 3D domain bounded by the Earth surface from above, by impervious boundary from bellow and by some vertical planes at the sides. Equation (2.4) contains two unknown functions, θ and ϕ, respectively, so it is desirable to find a relation between them in order to get one equation for one space - time unknown function. Such an equation together with proper boundary and initial conditions will give unique solution describing real groundwater flow situations. 3D domain is usually divided into a structure of aquifers and aquitards. Among the aquifers the upper one has the phreatic surface representing the boundary of the saturated zone called also as water table. Between the water table and the Earth surface is the so-called unsaturated zone. In this study we are interested in determining the phreatic surface - upper boundary of the saturated zone in the left floodplain area of Danube River. We consider only one aquifer representing the saturated zone with the lower (fixed) impervious boundary $z = \eta(x, y, t)$ and upper (free) boundary $z = \xi(x, y, t)$. In the saturated zone, the specific storativity is defined by the relation

$$s = \frac{d\theta}{d\phi}\,. \tag{2.5}$$

Using (2.5) we get the basic 3D equation of saturated flow in porous media

$$s\frac{\partial \phi}{\partial t} - \text{div } (k \nabla \phi) + w = 0\,. \tag{2.6}$$

In case of horizontally dominant flow (Dupuit assumption) in saturated aquifer this 3D equation can be reduced to 2D problem [4]. Introducing a piezometric head h as a mean value of the hydraulic potential with respect to vertical z - coordinate, i.e.

$$h(x, y, t) = \frac{\int_\eta^\xi \phi(x, y, z, t)\, dz}{\xi - \eta} \tag{2.7}$$

and integrating the equation (2.6) between η and ξ one gets (see [4]) 2D Boussinesq equation for the piezometric head h

$$\mu \frac{\partial h}{\partial t} = \frac{\partial}{\partial x}\left(k_{xx}(h - \eta)\frac{\partial h}{\partial x}\right) + \frac{\partial}{\partial y}\left(k_{yy}(h - \eta)\frac{\partial h}{\partial y}\right) - W \tag{2.8}$$

where $\mu = s(h - \eta) + n$ is a specific yield coefficient, n is porosity, k_{xx}, k_{yy} are coefficients of hydraulic conductivity in x and y directions, W is an averaged abstraction over the thickness of the aquifer. As a consequence of the Dupuit assumption the piezometric head h gives the z - coordinate of the water table as well. In this paper we have considered steady-state situation (unsteady model is an objective of our further research) and we have neglected the water abstraction term. Thus, we solve numerically the following steady nonlinear Boussinesq equation

$$-\frac{\partial}{\partial x}\left(k_{xx}(h - \eta)\frac{\partial h}{\partial x}\right) - \frac{\partial}{\partial y}\left(k_{yy}(h - \eta)\frac{\partial h}{\partial y}\right) = 0\,. \tag{2.9}$$

It is accompanied by Dirichlet boundary conditions for h. The details will be given in the next section.

3. Computational solution. The investigated area has a complicated geometry. Due to this fact, we have chosen FEM for discretization of 2D domain of interest (see Figures 1.1, 4.1 and 4.2). In order to be as closed as possible to the real geometrical situation we scanned the map of the left floodplain area of Danube River, loaded this map into AUTOCAD software and extracted precise lines representing boundary of the domain and the branch system (see Fig. 4.1). The FEM discretazion respects all these geometrical structures (see Fig. 4.2). For FEM discretization of PDE (2.9) we have used its analogy with the nonlinear heat transfer equation

$$-\frac{\partial}{\partial x}\left(K_{xx}(T)\frac{\partial T}{\partial x}\right)-\frac{\partial}{\partial y}\left(K_{yy}(T)\frac{\partial T}{\partial y}\right)=0\,. \tag{3.1}$$

where T represents the temperature and K_{xx}, K_{yy} thermal conductivity coefficients depending on T. Solution of such nonlinear elliptic PDE by FEM has been implemented in ANSYS software. In our case the coefficient of hydraulic conductivity is constant, i.e. $k = k_{xx} = k_{yy}$ and thus we have the following analogy $h \leftrightarrow T$ and $k(h-\eta) \leftrightarrow K_{xx}(T) = K_{yy}(T)$. The impervious boundary η is given by geological measurement. We provided a simplification dividing computational domain into the thirteen subdomains for which we put different constants (mean geodetic heads) for η. It would be rather complicated to consider precise η in each element (the subdomains are also visible in Fig. 4.1). Concerning the Dirichlet boundary conditions for h we consider as prescribed values the surface water levels in the Danube River bed, seepage canal, downstream power canal and in the branch system depending on discharges and on scenarios. Determination of surface water level regime in the branch system is based on the results of research group from Water Research Institute in Bratislava [5], who were concentrated in physical modelling of surface water level in the branch system at different discharges. Discretizing (3.1) by FEM, ANSYS software gets a nonlinear system of equations, which is then solved by the Newton - Raphson iterative method.

4. Discussion on numerical results. Scenarios of solution. Numerical solution was performed for two scenarios. They differ one from each other by constructing of hydraulic structures in the Danube River. The first scenario represents the current state, i.e., discharge in Danube River is 200 $m^3.s^{-1}$ and there are no structures in the river bed built, the discharges in the branch system and downstream power canal are 28 $m^3.s^{-1}$ and 1500 $m^3.s^{-1}$, respectively. According to results illustrated in Fig. 4.3 and Fig. 4.5, respectively, water levels in the river branch system are dominant for the groundwater flow simulation. It is given by the artificial water supply which has to eliminate the drainage effect of lower water levels in the Danube. The effect of this water supply into the branch system is neglected in the southern part of the floodplain where no branches exist. The second scenario was solved for a forecast state, i.e., after construction of designed underwater weirs in Danube River bed, with same discharges in Danube River (200 $m^3.s^{-1}$), branch system (28 $m^3.s^{-1}$) and downstream power canal (1500 $m^3.s^{-1}$). From these results (Fig. 4.4 and Fig. 4.6) follows that the water supply effect in the river branch system is still high and it is supported by increased water levels in the Danube. Therefore is the drainage effect of the Danube smaller, especially in the southern part. The same situation can be seen in the northern part, as well, where the effects of technical measures have been coupled. Both situations are very well illustrated by vector field of filtration velocities in this area.

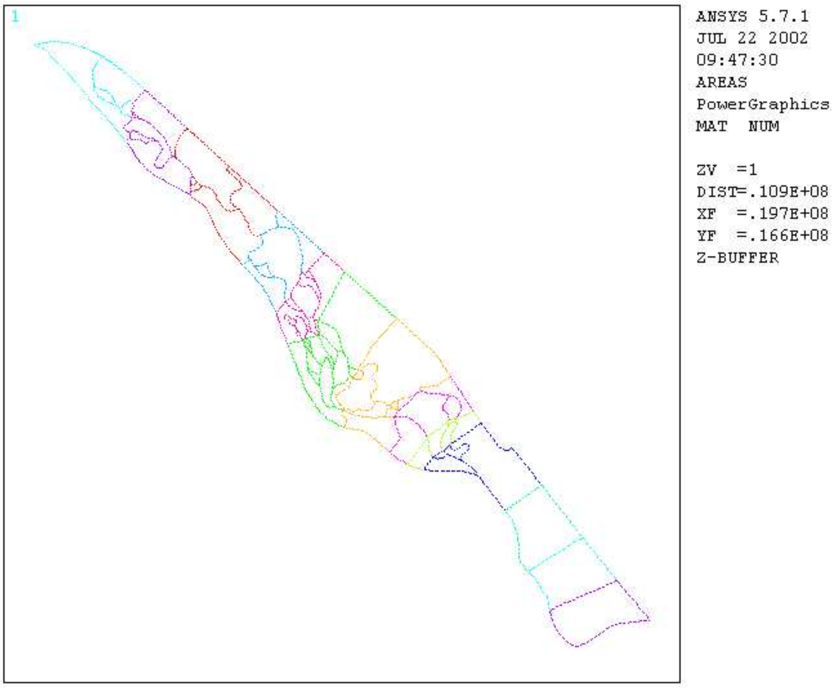

FIG. 4.1. *Geometrical model for FEM analysis of the area.*

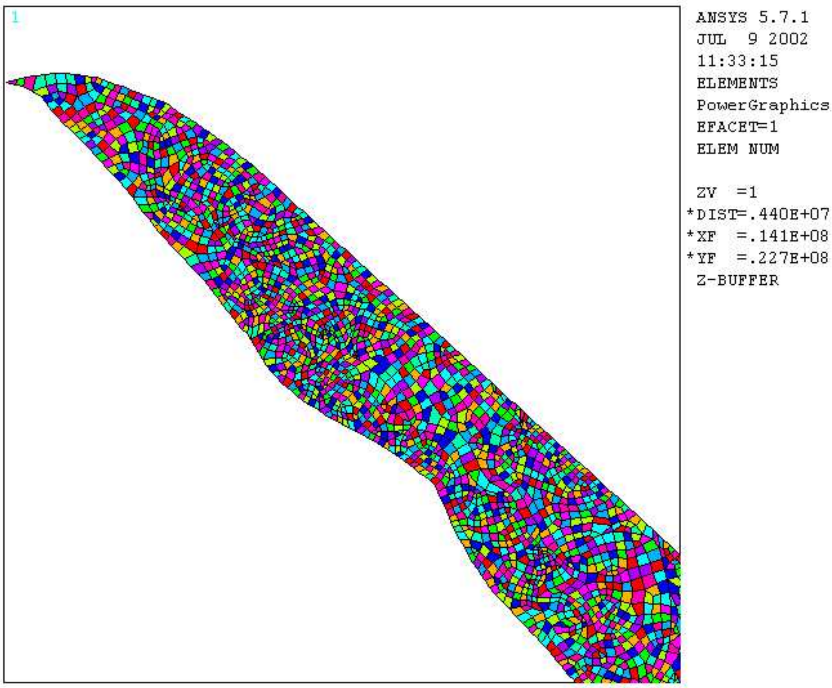

FIG. 4.2. *Discretization of the domain (we show only the upper part).*

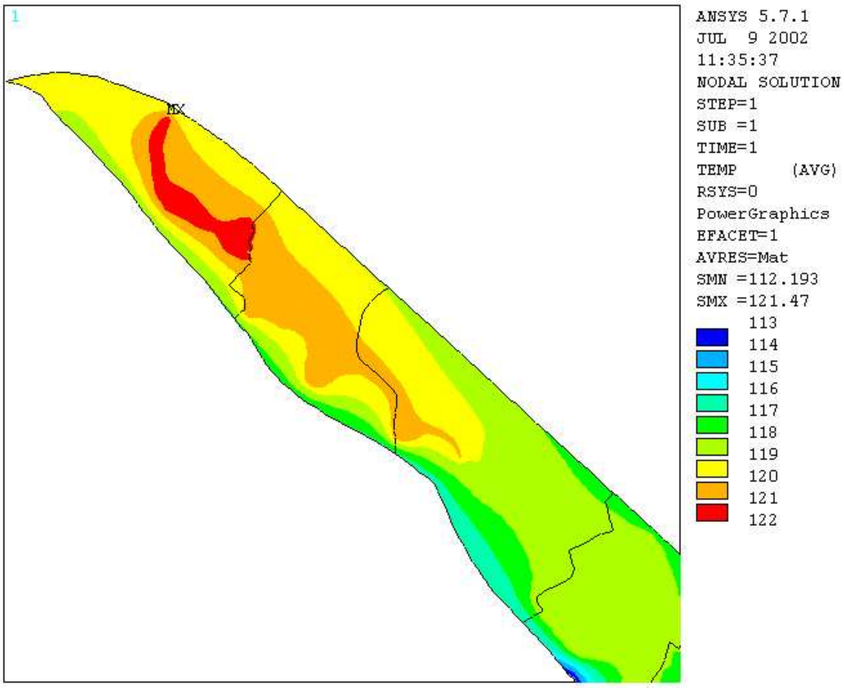

FIG. 4.3. *Groundwater levels without any technical measures undertaken in the Danube River - Scenario 1.*

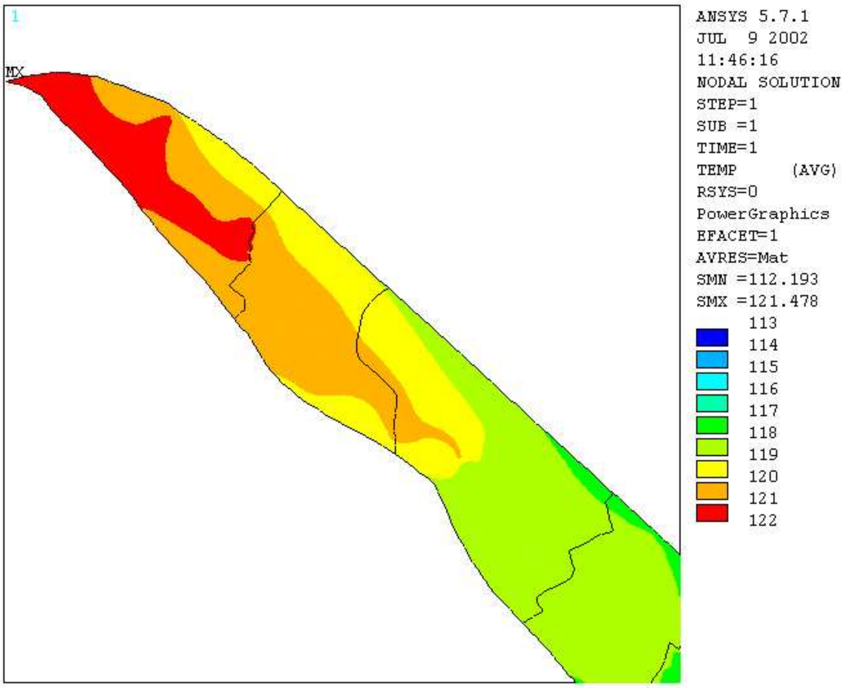

FIG. 4.4. *Groundwater levels with underwater weirs in the Danube River - Scenario 2.*

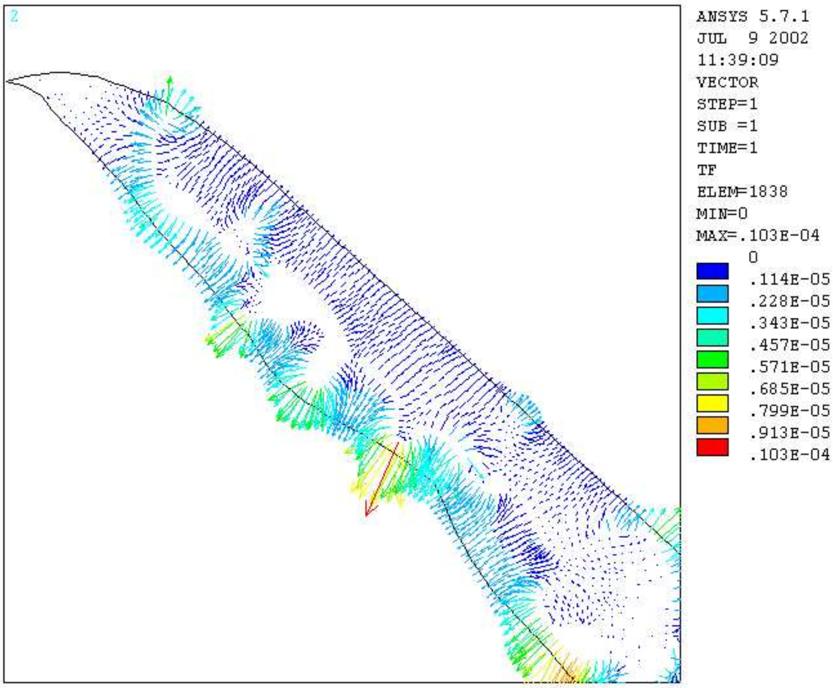

FIG. 4.5. *Vector field of filtration velocities - Scenario 1.*

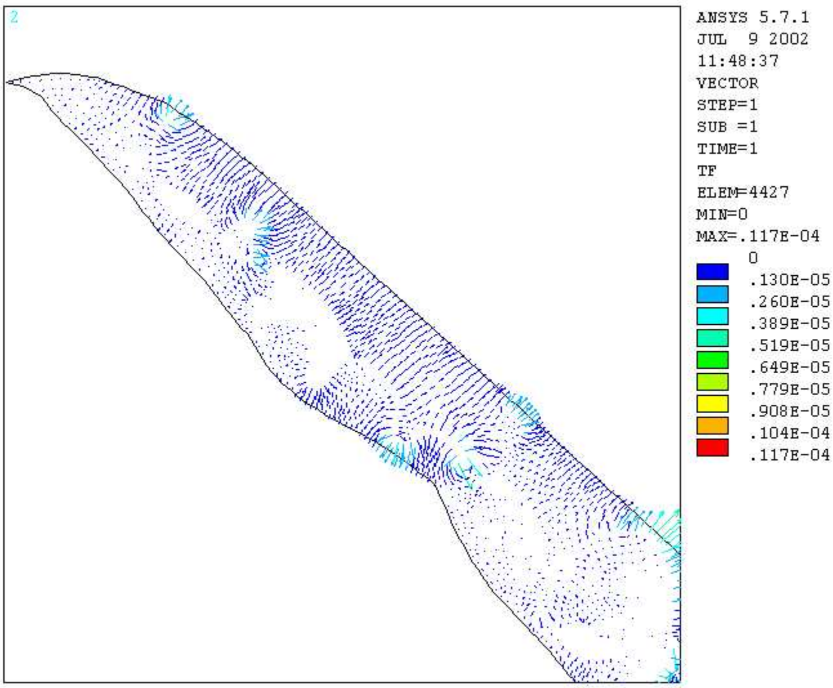

FIG. 4.6. *Vector field of filtration velocities - Scenario 2.*

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