1. cvičenie

Komplexné čísla

① a) $i^{134} = i^1 = i$

$i^1 = i = \sqrt{-1}$ $\qquad 134 \bmod 4 = 1$

$i^2 = i^1 \cdot i = (\sqrt{-1})^2 = -1$

$i^3 = i^2 \cdot i = -1 \cdot i = -i$

$i^4 = i^3 \cdot i = -i \cdot i = -i^2 = -(-1) = 1$

$i^5 = i^4 \cdot i = 1 \cdot i = i$

b) $i^{243} = i^3 = -i$

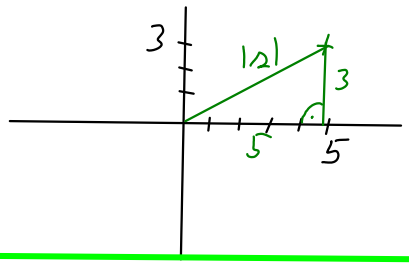
$243 \bmod 4 = 3$

---

②

$z = 5 + 3i$

$|z| = \sqrt{25+9} = \sqrt{34}$



---

③

$u = 2 + 3i$

$v = 1 - 2i$

$u + v = 3 + i$

$u - v = 1 + 5i$

---

(4) $u = 3+2i$

$v = 7-8i$

$u \cdot v = (3+2i)(7-8i) = 21+14i-24i-16i^2 = 37-10i$ ($i^2 = -1$)

$\frac{u}{v} = \frac{(3+2i)}{7-8i} \cdot \frac{(7+8i)}{7+8i} = \frac{21+14i+24i+16i^2}{49-64i^2} = \frac{5+38i}{113} = \frac{5}{113} + \frac{38}{113}i$ ($\frac{(7+8i)}{7+8i} = 1$)

$(a+b)(a-b) = a^2-b^2$

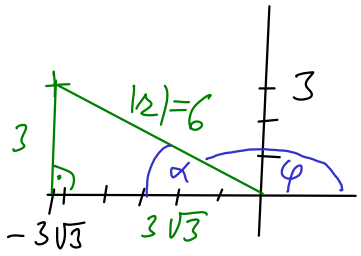
---

(5) a) $z = -3\sqrt{3}+3i$

$|z| = \sqrt{9 \cdot 3 + 9} = \sqrt{36} = 6$

$\sin\alpha = \frac{3}{6} = \frac{1}{2}$

$\alpha = 30° = \frac{\pi}{6} \Rightarrow \varphi = \frac{5}{6}\pi$



$\alpha + \varphi = 180°$

$z = |z|(\cos\varphi + i\sin\varphi)$

$z = 6\left(\cos\frac{5}{6}\pi + i\sin\frac{5}{6}\pi\right)$

b) $z = 3(\cos 120° + i\sin 120°) = -\frac{3}{2} + \frac{3\sqrt{3}}{2}i$

$\cos 120° = -\frac{1}{2}$

$\sin 120° = \frac{\sqrt{3}}{2}$

---

(6) $u = 2(\cos 45° + i\sin 45°)$

$v = 3(\cos 15° + i\sin 15°)$

$u \cdot v = 6(\cos 60° + i\sin 60°) = 3 + 3\sqrt{3}i$

$\frac{u}{v} = \frac{2}{3}(\cos 30° + i\sin 30°) = \frac{\sqrt{3}}{3} + \frac{i}{3}$

---

(7) Moivrova veta

$$\sqrt[3]{64i} = (64i)^{\frac{1}{3}}$$

$$64i = 64\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right)$$

$$\sqrt[3]{64i} = \sqrt[3]{64}\left(\cos\frac{\frac{\pi}{2}}{3} + i\sin\frac{\frac{\pi}{2}}{3}\right) = 4\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right) = 4(\cos 30^\circ + i\sin 30^\circ) = \underline{2\sqrt{3} + 2i}$$

---

(8) $(2+2i)^{13}$

$z = 2+2i \qquad \varphi = 45^\circ = \frac{\pi}{4}$

$|z| = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$

$$2+2i = 2\sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$$

$$(2+2i)^{13} = (2\sqrt{2})^{13}\left(\cos\frac{13}{4}\pi + i\sin\frac{13}{4}\pi\right) =$$

$$= 2^{13}\cdot 2^6\cdot\sqrt{2}\left(\cos\frac{5}{4}\pi + i\sin\frac{5}{4}\pi\right) =$$

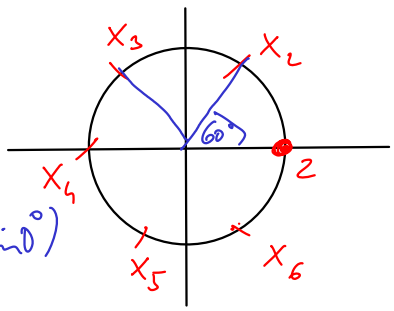
$$= 524288\sqrt{2}\left(\cos\frac{5}{4}\pi + i\sin\frac{5}{4}\pi\right)$$

---

(9) $x^6 - 64 = 0$

$x^6 = 64$

$x_1 = \sqrt[6]{64} = 2 = 2(\cos 0^\circ + i\sin 0^\circ)$



$$x_i = 2\left(\cos\frac{k}{3}\pi + i\sin\frac{k}{3}\pi\right), \quad k \in \{0,1,2,3,4,5\}$$

---

(10) $5x^3 - 11x^2 + 11x - 5 = 0$

$\{\pm 1, \pm 5\}$

$x_1 = 1$

$x_1 = 1$

$(5x^3 - 11x^2 + 11x - 5) : (x - 1) = 5x^2 - 6x + 5$

$-(5x^3 - 5x^2)$

$-6x^2 + 11x - 5$

$-(-6x^2 + 6x)$

$5x - 5$

$-(5x - 5)$

0

$5x^2 - 6x + 5 = 0$

$x_{2,3} = \frac{6 \pm \sqrt{36 - 100}}{10} =$

$= \frac{3}{5} \pm \frac{\sqrt{-64}}{10} =$

$= \frac{3}{5} \pm \frac{4}{5} i$

$x_1 = 1$

$x_2 = \frac{3}{5} + \frac{4}{5} i$

$x_3 = \frac{3}{5} - \frac{4}{5} i$