Singular points of algebraic varieties and some of their properties

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We consider a concept of algebraic varieties as a tool to capture the set of roots of a system of polynomial equations. The necessary notions from algebraic geometry and commutative algebra are briefly recalled. Their geometric interpretation is discussed. The local properties of singular points are formulated via the properties of local ring of variety at the singularity. Additional topics such as deformations of singularities are formulated on intuitive level with glimpses of necessary notions for rigorous continuation.

The concrete examples of isolated singularities are given on complex algebraic curves using Puiseux expansion and using the intersection multiplicity counting (algebraic and geometric). A knot associated with a singularity is considered as an invariant of the isolated singularity. We visualize the property of cabling including generating carousel construction and give several examples with Puiseux characteristic of length up to 3.

The topic of multifocal lemniscates in Euclidean plane is discussed. We survey basic properties of the families of confocal lemniscates. The singularities of such families give very important topological properties of the curves. They can be computed as roots of the defining complex polynomials. The location is described via Gauss-Lucas theorem. An application in reconstruction techniques is planned.

Space-like conics in Minkowski space are classified (done with PhD. student Barbora Pokorná). We give a glimpse of necessary and sufficient conditions for a Bézier curve to be space-like based on the characterization of intersection multiplicity. The work continues with higher degree plane curves.