The use of fuzzy logic in Location problems

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- Three types:
    - Maximal covering location problem (MCLP),
    - Minimal covering location problem (MinCLP)and
    - Location set covering problem (LSCP).

Covering location problem and Fuzzy covering location problem

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- Problem: Real problems contain some degree of uncertainty:
  - eg. Covering radius is about 5 kilometers or distance is between 8 and 10 kilometers.
- Solution: Fuzzy sets
  - Fuzzy number for radius of coverage.

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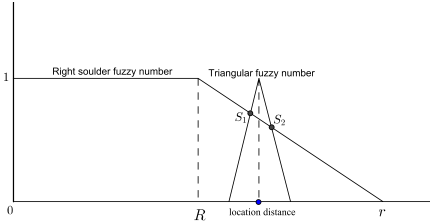

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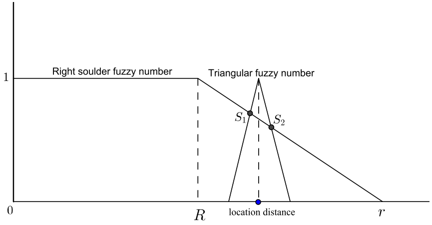
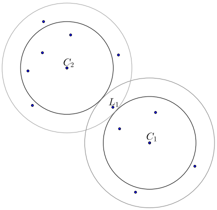


- Degree of coverage = arithmetic mean of fuzzy values S_1 and S_2.

Intersection of fuzzy radii

- What is degree of location covered with several fuzzy radii?



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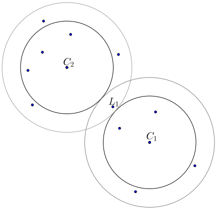

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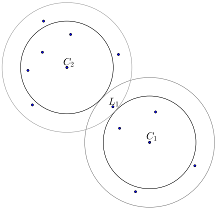

- Conorms:
  - Limited sum
    - it is allowed to sum coverage degrees
  - Maximal fuzzy value
    - it is NOT allowed to sum coverage degrees

Example

Limited sum

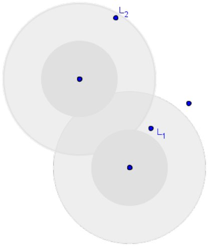

Figure 1: Optimal solution for MinCLP with limited sum conorm, Result $= 2.9$

Example

Maximal fuzzy value

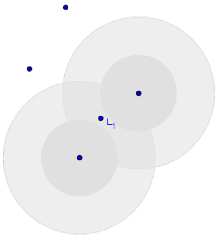

Figure 2: Optimal solution for MinCLP with maximal fuzzy value, Result = 2.8

Mathematical model of M(ax)CLP

Notation:

- P - number of facilities [integer]
- D_{ij} - distance matrix - (d_{ij}) is distance between i and j nodes [real]
- f_d - fuzzy distance value [real]
- $R + f_r$ - coverage radius [right shoulder fuzzy number].
- A_{ij} - coverage matrix - (a_{ij}) is degree of coverage node j by facility on node i (intersection of fuzzy numbers) [real]
- y_i - degree of coverage of location i $x_i \in [0, 1]$,
- x_i - indicator if a facility is established in the node i, $y_i \in \{0, 1\}$,

Mathematical model of M(ax)CLP - Limited sum conorm

Maximize

$$\sum_i y_i \tag{1}$$

with conditions:

$$\sum_j (x_j \cdot A_{ij}) \geq y_i, \forall i \tag{2}$$

$$x_i \cdot D_{i,j} > m, \forall j \tag{3}$$

$$\sum_i x_i = P \tag{4}$$

$$x_i \in \{0, 1\} \tag{5}$$

$$y_i \in [0, 1] \tag{6}$$

Mathematical model of M(ax)CLP - Maximal fuzzy number conorm

Maximize

$$\sum_i y_i \tag{7}$$

with conditions:

$$\max_j (x_j \cdot A_{ij}) \geq y_i, \forall i \tag{8}$$

$$x_i \cdot D_{i,j} > m, \forall j \tag{9}$$

$$\sum_i x_i = P \tag{10}$$

$$x_i \in \{0, 1\} \tag{11}$$

$$y_i \in [0, 1] \tag{12}$$

Solving MinCLP

- IBM CPLEX solver
  - Limited sum conorm: up to 1000 nodes, very slow for larger dimension,
  - Maximal fuzzy value conorm: up to 90 nodes

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- IBM CPLEX solver
  - Limited sum conorm: up to 1000 nodes, very slow for larger dimension,
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- Own-developed algorithm:
  - Based on Particle swarm optimization (PSO) metaheuristic

Particle swarm optimization (PSO) metaheuristic

- Nature-based metaheuristic method, introduced by Kennedy and Eberhart in 1995.
- Inspired by social behaviour of particles in swarms, like birds in flocks
- Instances (particles) are moving through solution space with some given intelligence: each particle knows its best position so far and the best position of its neighbourhood, and updates it own position using this information

Tests

- Generated instances: locations are randomly set in 30x30 grid
- Swarms contain 10 particles
- Dimensions up to 900 locations
- Numbers of facilities $P = 50, 80$
- Fuzzy radius of coverage $R + f_R = 2 + 0.5$
- Fuzzy distance value $f_d = 0.1$

Computational results

n	p	r	Fr	Fd	CPLEX P	v	PSD P	v
50	10	5	0.5	0.1	45.83	10587	45.83	470
80	10	0.5	0.1	45.83	N/A	10^8	76.81	1825

Thank you for your attention