A class of internal fusion operators for blind image filtering

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Motivation

The need to aggregate several values into a single value arises in almost every application.

Properties of aggregation functions for specific application:

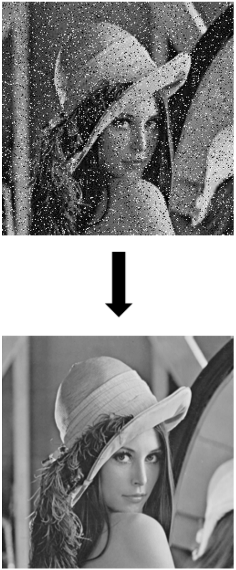
- Do we need every basic property?
- De we need some extra properties?

Sometimes it is important that the resulting output does not incorporate any new information from that already contained in the inputs.

Example: Image processing: **filtering** or reducing images.

Impulsive noise

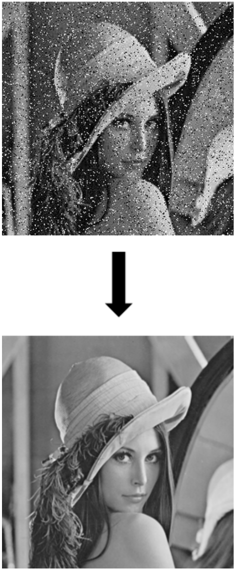
Impulsive noise



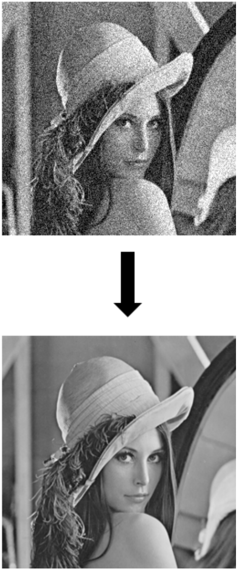
Impulsive noise

Additive noise

Impulsive noise



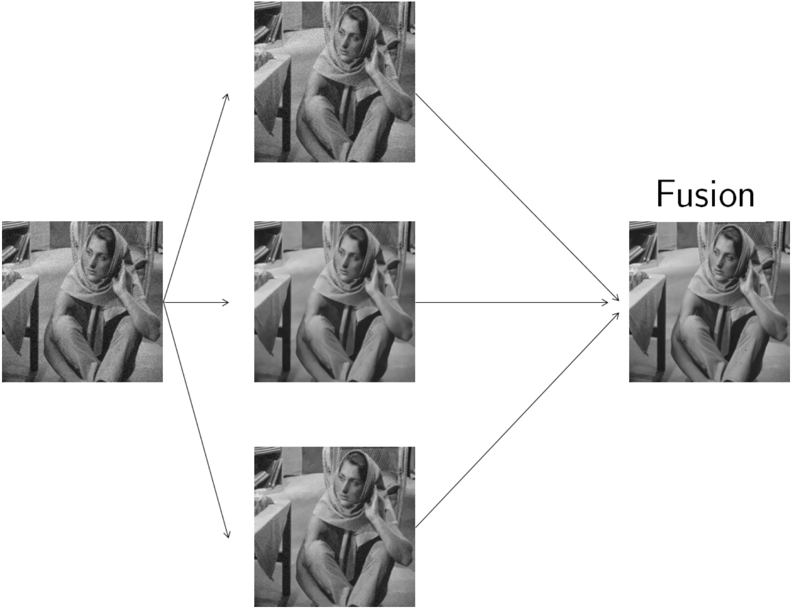
Additive noise

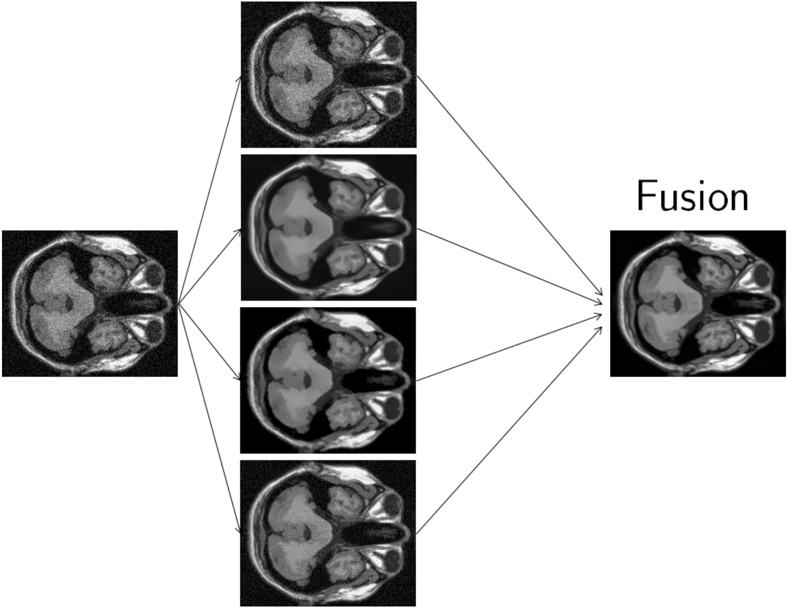


Impulsive noise

Additive noise

Any other noise...
Which filter?


Fusion


Fusion

Starting point

Definition

An internal operator is a mapping $F : [0,1]^n \to [0,1]$ such that

$$F(x_1, \ldots, x_n) \in \{x_1, \ldots, x_n\}$$

for every $(x_1, \ldots, x_n) \in [0,1]^n$.

Notice that only internality is demanded in our definition, but not monotonicity.

Starting point

It is clear that boundary conditions are satisfied.

Proposition

Let F be an internal operator. The following items hold:

i) $F(x, \ldots, x) = x$ *for all* $x \in [0, 1]$;

ii) $min(x_1, \ldots, x_n) \leq F(x_1, \ldots, x_n) \leq max(x_1, \ldots, x_n)$ *for all* $(x_1, \ldots, x_n) \in [0, 1]^n$.

Some examples

Example

1. Let π_j denote the j-th projection given by

$$\pi_j(x_1, \ldots, x_n) = x_j$$

   Then, for every $j \in \{1, \ldots, n\}$, the operator π_j is an internal operator.
2. Both the minimum and the maximum are internal operators.
3. Mode

Previous definition

Definition

$f : [0, 1]^n \to [0, 1]$ is a locally internal aggregation function if:

1. f is continuous.
2. f is non-decreasing.
3. $f(x, \ldots, x) = x$ for every $x \in [0, 1]$.
4. f is locally internal; that is, $f(x_1, \ldots, x_n) \in \{x_1, \ldots, x_n\}$ for every $(x_1, \ldots, x_n) \in [0, 1]^n$.

G. Mayor and J. Martin, Locally internal aggregation functions, International Journal of Uncertainty, Fuzziness and Knowledge-based Systems, 7 (1999) 235–241.

Construction of internal aggregation functions

Theorem

Let $M : [0,1]^n \to [0,1]$ *be an aggregation function and let* $a_0, a_1, \dots, a_n \in [0,1]$ *such that* $0 = a_0 \leq a_1 \leq \cdots \leq a_n = 1$. *The mapping* $F_{M,a} : [0,1]^n \to [0,1]$ *given by*

$$F_{M,a}(x_1, \dots, x_n) = \begin{cases} x_{(1)} & \text{if } M(x_1, \dots, x_n) = 0 \\ x_{(i)} & \text{if } M(x_1, \dots, x_n) \in]a_{i-1}, a_i] \end{cases}$$

is an internal aggregation function.

Proposition

The following items hold

i) *Let* $a_1 = a_2 = \cdots = a_n = 1$. *Then*

$$F_{M,a}(x_1, \ldots, x_n) = \min(x_1, \ldots, x_n)$$

for every aggregation function M *and* $(x_1, \ldots, x_n) \in [0,1]^n$;

ii) *Let* $0 = a_0 = a_1 = \cdots = a_{n-1}$. *Then*

$$F_{M,a}(x_1, \ldots, x_n) = \max(x_1, \ldots, x_n)$$

for every aggregation function M *and* $(x_1, \ldots, x_n) \in [0,1]^n$;

ii) *Let* n *be odd and* $0 = a_0 = a_1 = \cdots = a_{\frac{n-1}{2}}$ *and* $a_{\frac{n+1}{2}} = \cdots = a_n = 1$. *Then*

$$F_{M,a}(x_1, \ldots, x_n) = median(x_1, \ldots, x_n)$$

for every aggregation function M *and* $(x_1, \ldots, x_n) \in [0,1]^n$.

Definition

Let $P : [a, b]^{n+1} \to \mathcal{R}$ be a penalty function with the properties

i) $P(\mathbf{x}, y) \geq 0$ for all $\mathbf{x}, y$;

ii) $P(\mathbf{x}, y) = 0$ if all $x_i = y$;

iii) $P(x, y)$ is quasi-convex in y for any $\mathbf{x}$.

The penalty based function is

$$f(\mathbf{x}) = \arg\min_y P(\mathbf{x}, y)$$

if y is the unique minimizer, and $y = \frac{a+b}{2}$ if the set of minimizers is the interval $[a, b]$.

Theorem

Calvo et al. Any averaging aggregation function can be expressed as a penalty based function.

Theorem

Let $P : [0,1]^{n+1} \to [0,1]$ *be a penalty function. Let* $F_P : [0,1]^n \to [0,1]$ *be given by*

$$F_P(x_1, \ldots, x_n) = \arg\min_{x_i} P(\mathbf{x}, x_i) \text{ for all } i = 1, ..., n$$

Then F_P *is an internal operator.*

Monotonicity? Let $P(\mathbf{x}, y) = \sum_{i=1}^{n} (x_i - y)^2$
$F_P(0, 0.1, 0.3, 0.41) = 0.3$
$F_P(0, 0.2, 0.3, 0.41) = 0.2$

Following previous example: let M be the arithmetic mean.
$M(0, 0.1, 0.3, 0.41) = 0.2025$. Closest value is $0.3 = F_P(0, 0.1, 0.3, 0.41)$
$M(0, 0.2, 0.3, 0.41) = 0.2275$. Closest value is $0.2 = F_P(0, 0.2, 0.3, 0.41)$

Definition

The mapping $d_R : [0,1]^2 \to [0,1]$ is a restricted dissimilarity function if it satisfies:

(1) $d_R(x,y) = d_R(y,x)$ for every $x, y \in [0,1]$;

(2) $d_R(x,y) = 1$ if and only if $\{x,y\} = \{0,1\}$;

(3) $d_R(x,y) = 0$ if and only if $x = y$;

(4) For any $x, y, z \in [0,1]$, if $x \leq y \leq z$, then $d_R(x,y) \leq d_R(x,z)$ and $d_R(y,z) \leq d_R(x,z)$.

Theorem

Bustince et al. Take $d_R : [0,1]^2 \to [0,1]$. Then the following items are equivalent:

(i) *d_R is a faithful restricted dissimilarity function.*

(ii) *There exists a convex automorphism φ and a bijection h on the unit interval such that $d_R(x,y) = \varphi(|h(x) - h(y)|)$ for all $x, y \in [0,1]$.*

Proposition

Bustince et al. Let $d_R : [0,1]^2 \to [0,1]$ *be a faithful dissimilarity function. Then the function* $P : [0,1]^{n+1} \to [0,1]$ *defined for any* $x_1, \ldots, x_n \in [0,1]$ *as*

$$P(x_1, \ldots, x_n) = \sum_{i=1}^{n} d_R(x_i, y)$$

is a penalty function.

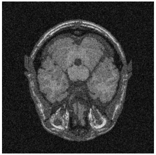
Proposition

Let d_R *be a restricted dissimilarity function as in previous Theorem with* $h(x) = x$ *for all* $x \in [0,1]$. *Let* P *be a penalty function given by* $P(x_1, \ldots, x_n, y) \sum_{i=1}^{n} d_R(x_i, y)$. *Then* $F_P(x_1 + a, \ldots, x_n + a) = F_P(x_1, \ldots, x_n) + a$ *for all* $x_1, \ldots, x_n, a \in [0,1]$

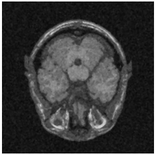
Experimental study

- Set of 50 medical images: original and corrupted with rician noise
- A set of 6 filters: 3 types of filters (median, mean and gaussian) with 2 different settings
- Internal operator based on penalty functions:
  - $P(x_1, \ldots, x_n) = \sum_{i=1}^{n} |x_i - y|$
  - $P(x_1, \ldots, x_n) = \sum_{i=1}^{n} (x_i - y)^2$

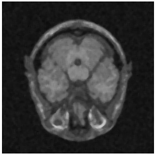
Experimental study



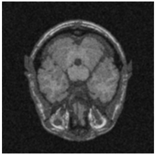
Original



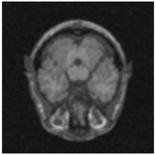
Filter 1



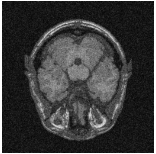
Filter 2



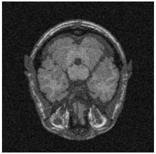
Filter 3



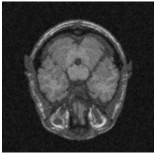
Filter 4



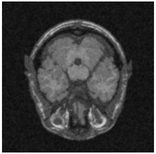
Filter 5



Filter 6



Fusion P1



Fusion P2

Experimental study

Comparison with original (without noise) image

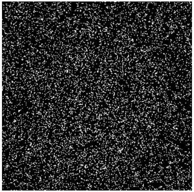
- Mean squared error
- Simmilarity measure based on contrast de-enhancement

Filter 1	Filter 2	Filter 3	Filter 4	Filter 5	Filter 6
323.8	379.0	343.5	412.9	337.4	337.3
0.9400	0.9375	0.9369	0.9328	0.9382	0.9382

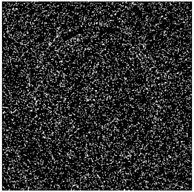
Fusion P1	Fusion P2
323.5	326.0
0.9390	0.9386

Pixels selected from each filter

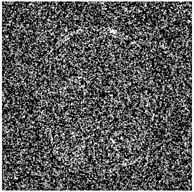
Using P2, the fusion image pixels are selected from:



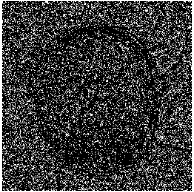
Filter 1



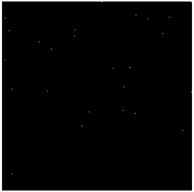
Filter 2



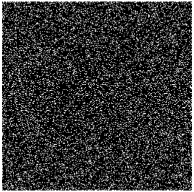
Filter 3



Filter 4



Filter 5



Filter 6

Experimental study

Image corrupted with gaussian noise

Med3	Med4	Avg3	Avg5	Gau3	Gau5
2128.8	2160.6	2147.9	2214.5	2140.4	2140.4
0.8318	0.8302	0.8306	0.8279	**0.8325**	0.8312

IntP1	IntP2	IntP3
2132.1	2132.3	**2128.4**
0.8312	0.8312	0.8313

Conclusions

Given an application of blind image fusion we have studied the concept of internal fusion operator

- Internal aggregation functions (continuos and non-continuous)
- Internal operators constructed from penalty functions

We have seen that the results obtained are promising. Our idea

- To continue studying properties of internal operators
- To use more sophisticated filters and observe the result obtained by intenal fusion

Thanks for your attention