Semantic Interpretation of Intermediate Quantifiers and Their Syllogisms with Non-trivial Conclusions

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OP Výzkum a vývoj pro inovace

Outline

What is a classical syllogism?

Definition of classical syllogism.

- A classical syllogism denoted by $\langle P_1, P_2, C\rangle$ is a kind of logical argument in which the *conclusion* C is inferred from two *premises* — *major* P_1 and *minor* P_2.

Aristotle's syllogisms.

- All Aristotle's syllogisms work with the classical quantifiers "All", "No", "Some".

Intermediate syllogisms.

- By intermediate syllogism we mean traditional syllogism where we replace one or more of its formulas with some containing intermediate quantifiers.
- "Almost all", "Most", "Many".

Classification of syllogisms

Suppose that Q_1, Q_2, Q_3 are intermediate quantifiers and X, Y, M are formulas.

Figure I

$$\frac{\begin{array}{c} Q_1\ X \text{ is } M \\ Q_2\ M \text{ is } Y \end{array}}{Q_3\ X \text{ is } Y}$$

Figure II

$$\frac{\begin{array}{c} Q_1\ Y \text{ is } M \\ Q_2\ X \text{ is } M \end{array}}{Q_3\ X \text{ is } Y}$$

Figure III

$$\frac{\begin{array}{c} Q_1\ M \text{ is } Y \\ Q_2\ M \text{ is } X \end{array}}{Q_3\ X \text{ is } Y}$$

Figure IV

$$\frac{\begin{array}{c} Q_1\ Y \text{ is } M \\ Q_2\ M \text{ is } X \end{array}}{Q_3\ X \text{ is } Y}$$

Classification of syllogisms

If we work with five basic intermediate quantifiers (“All”, “Almost all”, “Most”, “Many” and “Some”) we obtain more than 2000 possibilities of syllogisms.

Valid syllogisms

- 24 Aristotle’syllogisms are valid.
- 81+24 Intermediate syllogisms are valid.
- We syntactically verified the validity of all 105 syllogisms in the theory of intermediate quantifiers.

Examples of Syllogisms of Figure-I

Barbara-AAA-I

P_1: All Greeks are men.
P_2: All men are mortal.

C: All Greeks are mortal.

AAT-I

P_1: All Greeks are men.
P_2: All men are mortal.

C: Most Greeks are mortal.

Examples of Syllogisms of Figure-I

One premise is an universal-TAT-I

P_1: Most Greeks are men.
P_2: All men are mortal.

C: Most Greeks are mortal.

Both premises are non-universal

P_1: Most Greeks are men.
P_2: Most men are mortal.

C: ????? Greeks are mortal.

Relationships with other authors

- These syllogisms were studied by Zadeh's, Peterson's, Dubois's, Fariña and many others.
- Peterson verified the validity using Venn Diagram but only for particular conclusion (Some).
- Other authors semantically verified (with special assumptions) the validity of this syllogism with non-trivial conclusion.
- We proposed mathematical formal system which is higher order fuzzy logic, namely Łukasiewicz fuzzy type theory, based on Łukasiewicz MV-algebra.
- The main idea is to use formal mathematical system and syntactically and semantically verify the validity of this syllogism.

Formal mathematical system

- We have four figures and it means that we have 105 intermediate generalized syllogisms which are valid.
- We proposed mathematical formal system which is higher order fuzzy logic, namely Łukasiewicz fuzzy type theory, based on Łukasiewicz MV-algebra.
- We developed the theory of intermediate quantifiers which has 17 axioms and two deduction rules.

Definition of intermediate generalized quantifiers

We can define all intermediate quantifiers in higher order fuzzy logic as follows:

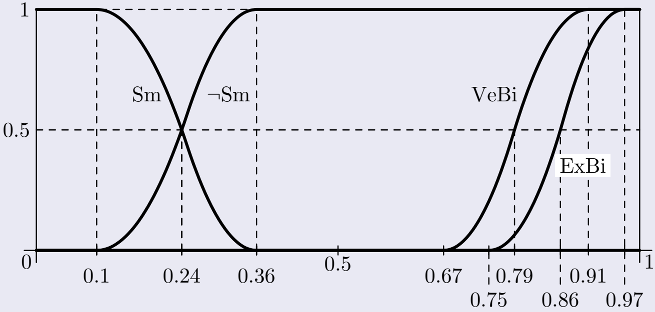
Explanation of definition of IGQ

$$\underbrace{(\exists z)((\boldsymbol{\Delta}(z \subseteq B)}_{\text{“the greatest” part of } B\text{'s}} \quad \& \quad \underbrace{(\forall x)(z\,x \Rightarrow Ax))}_{\text{each } z\text{'s has } A} \quad \wedge \quad \underbrace{Ev((\mu B)z))}_{\text{size of } z \text{ is evaluated by } Ev} \tag{1}$$

We can define the intermediate quantifiers Almost all, Most, Many, Few.

Illustration of interpretation evaluative expressions

Extensions of evaluative expressions



Shapes of extensions of evaluative expressions in the context $\langle 0, 0.5, 1\rangle$

Validity of syllogism

Syntactical definition

- The syllogism is valid if $T^{\mathrm{IQ}} \vdash P_1 \,\&\, P_2 \Rightarrow C$, or equivalently, if $T^{\mathrm{IQ}} \vdash P_1 \Rightarrow (P_2 \Rightarrow C)$.

Semantical definition

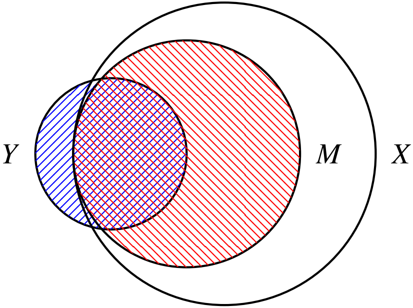
- The syllogism is valid in the theory T^{IQ} if there is a model $\mathcal{M} \models T^{\mathrm{IQ}}$ such that $\mathcal{M}(P_1) \otimes \mathcal{M}(P_2) \leq \mathcal{M}(C)$.

Examples of Syllogisms of Figure-I

One premise is an universal

P_1: Most Greeks (Y) are men (M).
P_2: All men (M) are mortal (X).

C: Most Greeks (Y) are mortal (X).



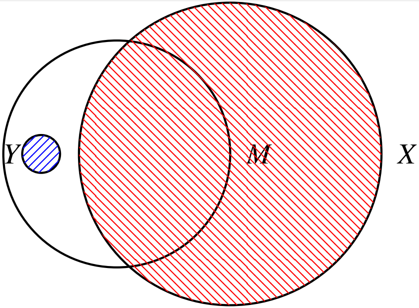
This syllogism is strongly valid in every model.

Examples of Syllogisms of Figure-I

Both premises are intermediate

P_1: Most Greeks (Y) are men (M).
P_2: Most men (M) are mortal (X).

C: Many Greeks (Y) are mortal (X).



If $\mathcal{M}(P_1) = 1$ and $\mathcal{M}(P_2) = 1$ then $\mathcal{M}(C) < 1$. It means that this syllogism is invalid.

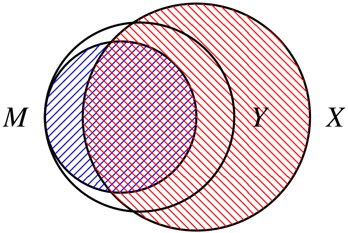
Examples of Syllogisms of Figure-I

Let $M \subseteq Y$.

Both premises are intermediate

P_1: Most Greeks (Y) are men (M).
P_2: Most men (M) are mortal (X).

C: Many, Some Greeks (Y) are mortal (X).



Results

Syntactical

Let T^{IQ} be a theory. Let X, Y, M be a normal fuzzy sets such that $M \subseteq Y$. Then the following syllogism is strongly valid in T^{IQ}.

$$\begin{array}{l} P_1\text{: Most Greeks (Y) are men (M).} \\ P_2\text{: Most men (M) are mortal (X).} \\ \hline \text{C: Some Greeks (Y) are mortal (X).} \end{array}$$

The syllogism is as follows:

$$\begin{array}{l} P_1 : (\exists z)((\Delta z \subseteq Y)\, \&\, (\forall x)(zx \Rightarrow Mx) \wedge (Bi\ \mathrm{Ve})((\mu Y)z)). \\ P_2 : (\exists z')((\Delta z' \subseteq M)\, \&\, (\forall x)(z'x \Rightarrow Xx) \wedge (Bi\ \mathrm{Ve})((\mu M)z')). \\ \hline C : (\exists x)(Yx \wedge Xx). \end{array}$$

It means that we know find the proof of the formula
$T^{\mathrm{IQ}} \vdash P_1\, \&\, P_2 \Rightarrow C$.

Results

Semantical

Let T^{IQ} be a theory and $\mathcal{M}$ be a model of T^{IQ}. Let X, Y, M be a normal fuzzy sets such that $M \subseteq Y$. Then the following syllogism is strongly valid in the model $\mathcal{M}$.

P_1: Most Greeks (Y) are men (M).
P_2: Most men (M) are mortal (X).

C: Many Greeks (Y) are mortal (X).

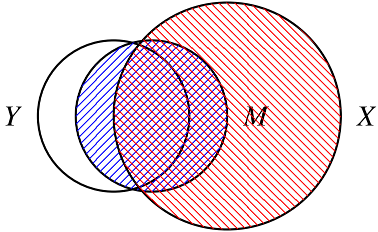
It means that if $\mathcal{M}(P_1) = a$ and $\mathcal{M}(P_2) = b$ then $a \otimes b \leq c = \mathcal{M}(C)$.

Example of Syllogism of Figure-I

Both premises are intermediate

P_1: Most Greeks (Y) are men (M).
P_2: Most men (M) are mortal (X).

C: Many Greeks (Y) are mortal (X).



Results

Semantical

Let T^{IQ} be a theory and $\mathcal{M}$ be a model of T^{IQ}. Let X, Y, M be a normal fuzzy sets such

- $T^{\mathrm{IQ}} \vdash \neg Sm\,\boldsymbol{\nu}\,\mu_Y(X \cap M \cap Y)$

holds. Then the following syllogism is strongly valid in the model $\mathcal{M}$.

P_1: Most Greeks (Y) are men (M).
P_2: Most men (M) are mortal (X).

C: Many Greeks (Y) are mortal (X).

It means that if $\mathcal{M}(P_1) = a$ and $\mathcal{M}(P_2) = b$ then $a \otimes b \leq c = \mathcal{M}(C)$.

Conclusions

- We introduced formal mathematical theory of intermediate quantifiers using Łukasiewicz fuzzy type theory.
- We proposed the definitions of intermediate quantifiers in our theory.
- We analyzed the syllogisms with itermediate quantifiers in both premises with non-particular conclusion.
- Construct non-finite models in the theory of intermediate quantifiers and analyze other type of syllogisms with itermediate quantifiers.

Thank You for Your Attention.