First-order EQ-logic with equality

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FSTA 2014, Liptovský Ján, Slovak Republic
January 26–31, 2014
First-order EQ-logic with equality

Outline

1. Motivation

2. EQ-algebras

3. Propositional EQ-logics
   - Basic EQ-logic
   - Extensions
   - Prelinear EQ_{\Delta}-logic

4. Predicate EQ-logic

5. Conclusion
Motivation

EQ-algebras

Propositional EQ-logics
  - Basic EQ-logic
  - Extensions
  - Prelinear $EQ_{\Delta}$-logic

Predicate EQ-logic

Conclusion
How did EQ-logic arise?

- Motivation comes from G. W. Leibniz, L. Wittgenstein and F. P. Ramsey. To develop logic on the basis of identity (equality) as the principle connective.

- Henkin’s type theory (higher ordered logic) was developed. [L. Henkin, A theory of propositional types, *Fundamenta Math.*, 52: 323–344, (1963).] A fully satisfactory logical calculus must be an equational one.”

First-order EQ-logic with equality

Motivation

How did EQ-logic arise?

How could fuzzy logic be developed on the basis of fuzzy equality?

- Residuated lattice \[ a \leftrightarrow b = (a \rightarrow b) \land (b \rightarrow a) \]

- EQ-algebra


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1 Motivation

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5 Conclusion
Non-commutative EQ-algebra is the algebra

\[ \mathcal{E} = \langle E, \land, \otimes, \sim, 1 \rangle \]

of type \((2, 2, 2, 0)\)

(E1) \( \langle E, \land, 1 \rangle \) is a commutative idempotent monoid (i.e. \( \land \)-semilattice with top element 1) with the ordering: \( a \leq b \) iff \( a \land b = a \)

(E2) \( \langle E, \otimes, 1 \rangle \) is a monoid and \( \otimes \) is isotone w.r.t. \( \leq \)
Definition

Non-commutative EQ-algebra is the algebra

\[ \mathcal{E} = \langle E, \wedge, \otimes, \sim, 1 \rangle \]

of type \((2, 2, 2, 0)\)

\((E1)\) \(\langle E, \wedge, 1 \rangle\) is a commutative idempotent monoid (i.e. \(\wedge\)-semilattice with top element 1) with the ordering: \(a \leq b\) iff \(a \wedge b = a\)

\((E2)\) \(\langle E, \otimes, 1 \rangle\) is a monoid and \(\otimes\) is isotone w.r.t. \(\leq\)
(E3) \( a \sim a = 1 \)  
(reflexivity)

(E4) \( ((a \land b) \sim c) \otimes (d \sim a) \leq c \sim (d \land b) \)  
(substitution)

(E5) \( (a \sim b) \otimes (c \sim d) \leq (a \sim c) \sim (b \sim d) \)  
(congruence)

(E6) \( (a \land b \land c) \sim a \leq (a \land b) \sim a \)  
(monotonicity)

(E7) \( a \otimes b \leq a \sim b \)  
(boundedness)

Implication: \( a \rightarrow b = (a \land b) \sim a \)
Definition (continued)

(E3) \( a \sim a = 1 \)  

(reflexivity)

(E4) \( ((a \land b) \sim c) \otimes (d \sim a) \leq c \sim (d \land b) \)  

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(monotonicity)

(E7) \( a \otimes b \leq a \sim b \)  

(boundedness)

Implication: \( a \to b = (a \land b) \sim a \)
### Special EQ-algebras

**EQ-algebra is**

1. **Good** if \( a \sim 1 = a \)
2. **Residuated** if
   \[
   (a \otimes b) \land c = a \otimes b \iff a \land ((b \land c) \sim b) = a
   \]
3. **Involutive** if \( \neg \neg a = a \) \hspace{1cm} (IEQ-algebra)
4. **Prelinear** if for all \( a, b \in E \) \( \sup\{a \rightarrow b, b \rightarrow a\} = 1 \).
5. **Lattice EQ-algebra** if it is a lattice-ordered and for all \( a, b, c, d \in E \)
   \[
   ((a \lor b) \sim c) \otimes (d \sim a) \leq (d \lor b) \sim c
   \] \hspace{1cm} (\ell EQ-algebra)
Special EQ-algebras

A lattice $\text{EQ}_\Delta$-algebra ($\ell\text{EQ}_\Delta$-algebra)

$\mathcal{E}_\Delta = \langle E, \land, \lor, \otimes, \sim, \Delta, 0, 1 \rangle$

- $\langle E, \land, \lor, \otimes, \sim, 0, 1 \rangle$ is a good non-commutative and bounded $\ell\text{EQ}$-algebra.

- $\Delta 1 = 1$
- $\Delta a \leq \Delta \Delta a$
- $\Delta (a \sim b) \leq \Delta a \sim \Delta b$
- $\Delta (a \land b) = \Delta a \land \Delta b$
- $\Delta a = \Delta a \otimes \Delta a$
- $\Delta (a \lor b) \leq \Delta a \lor \Delta b$
- $\Delta a \lor \neg \Delta a = 1$
- $\Delta (a \sim b) \leq (a \otimes c) \sim (b \otimes c)$
- $\Delta (a \sim b) \leq (c \otimes a) \sim (c \otimes b)$
Representation of $\ell\text{EQ}_\Delta$-algebras

Lemma

If a good EQ-algebra $\mathcal{E}$ satisfies

$$(a \rightarrow b) \lor (d \rightarrow (d \otimes (c \rightarrow ((b \rightarrow a) \otimes c)))) = 1$$

for all $a, b, c, d \in E$ then it is prelinear.

Theorem

Let $\mathcal{E}_\Delta$ be $\ell\text{EQ}_\Delta$-algebra. The following are equivalent:

(a) $\mathcal{E}_\Delta$ is subdirectly embeddable into a product of linearly ordered good $\ell\text{EQ}_\Delta$-algebras.

(b) $\mathcal{E}_\Delta$ satisfies (1).
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   - Prelinear EQΔ-logic

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5. Conclusion
Why EQ-logics

EQ-logics — special class of many-valued logics
truth values form an EQ-algebra

- Equivalence as the basic connective instead of implication
- Proofs in equational style
- Even more general than MTL-logics
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- Equivalence as the basic connective instead of implication
- Proofs in equational style
- Even more general than MTL-logics
Language

- Propositional variables $p_1, p_2, \ldots$
- Connectives: $\wedge$ (conjunction), $\&$ (fusion), $\equiv$ (equivalence),
- Logical constant $\top$ (true)

**Implication:**

$$A \Rightarrow B := (A \wedge B) \equiv A$$
First-order EQ-logic with equality

Propositional EQ-logics

Basic EQ-logic

Language

- Propositional variables $p_1, p_2, \ldots$
- Connectives: $\land$ (conjunction), $\&$ (fusion), $\equiv$ (equivalence),
- Logical constant $\top$ (true)

Implication:

$$A \Rightarrow B := (A \land B) \equiv A$$
Logical axioms

(EQ1) \((A \equiv \top) \equiv A\)

(EQ2) \(A \land B \equiv B \land A\)

(EQ3) \((A \cup B) \cup C \equiv A \cup (B \cup C), \quad \cup \in \{\land, \&\}\)

(EQ4) \(A \land A \equiv A\)

(EQ5) \(A \land \top \equiv A\)

(EQ6) \(A \& \top \equiv A\)

(EQ7) \(\top \& A \equiv A\)

(EQ8a) \(((A \land B) \& C) \Rightarrow (B \& C)\)

(EQ8b) \((C \& (A \land B)) \Rightarrow (C \& B)\)

(EQ9) \(((A \land B) \equiv C) \& (D \equiv A) \Rightarrow (C \equiv (D \land B))\) (substitution)

(EQ10) \((A \equiv B) \& (C \equiv D) \Rightarrow (A \equiv C) \equiv (B \equiv D)\) (congruence)

(EQ11) \((A \Rightarrow (B \land C)) \Rightarrow (A \Rightarrow B)\) (monotonicity)
Inference rules

**Equanimity rule**

*From* $A$ and $A \equiv B$ *infer* $B$

**Leibniz rule**

*From* $A \equiv B$ *infer* $C[p := A] \equiv C[p := B]$

$C[p := A]$ denotes a formula resulting from $C$ by replacing all occurrences of a variable $p$ in $C$ by the formula $A$. 
Semantics

**Truth values**
The set of truth values is a good non-commutative EQ-algebra $\mathcal{E} = \langle E, \land, \otimes, \sim, 1 \rangle$

**Theorem (Completeness)**
*For every formula $A \in F_J$ the following is equivalent:

(a) $\vdash A$

(b) $e(A) = 1$ for every truth evaluation $e : F_J \rightarrow E$ and every good non-commutative EQ-algebra $\mathcal{E}$.***
Other EQ-logics

- Involutive EQ-logic (with double negation)
- Prelinear EQ-logic (stronger variant of the completeness theorem)
- EQ(MTL)-logic (equivalent with MTL-logic)

Not strong enough for development of the predicate EQ-logic!

- Basic EQ\(\Delta\)-logic (weaker variant of the completeness theorem)
- Prelinear EQ\(\Delta\)-logic

Theorem (Deduction)

For each theory \(T\) and formulas \(A, B, C \in F_J\) :

\[ T \cup \{A \equiv B\} \vdash C \quad \text{iff} \quad T \vdash \Delta(A \equiv B) \Rightarrow C \]
First-order EQ-logic with equality

- Propositional EQ-logics
- Extensions

Other EQ-logics

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- Basic $EQ_\Delta$-logic (weaker variant of the completeness theorem)
- Prelinear $EQ_\Delta$-logic

**Theorem (Deduction)**

For each theory $T$ and formulas $A, B, C \in F_J$:

$T \cup \{A \equiv B\} \vdash C$ iff $T \vdash \Delta(A \equiv B) \Rightarrow C$
Other EQ-logics

- Involutive EQ-logic (with double negation)
- Prelinear EQ-logic (stronger variant of the completeness theorem)
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Not strong enough for development of the predicate EQ-logic!

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- Prelinear $EQ_\Delta$-logic

Theorem (Deduction)

For each theory $T$ and formulas $A, B, C \in F_J$:

$T \cup \{A \equiv B\} \vdash C$ iff $T \vdash \Delta(A \equiv B) \Rightarrow C$
Prelinear $E_{\Delta}$-logic

**Language**
The language of basic $E$-logic extended by unary connective $\Delta$, binary connective $\lor$ and logical constant $\bot$.

**Negation** $\neg A := A \equiv \bot$

Axioms (EQ1)–(EQ11) and

$(((A \land B) \lor C) \equiv D) \land (F \equiv C) \land (E \equiv A) \Rightarrow (D \equiv (F \lor (B \land E))))$

(EQ12) $(A \lor B) \lor C \equiv A \lor (B \lor C)$
(EQ13) $A \lor (A \land B) \equiv A$
(EQ14) $(A \land \bot) \equiv \bot$
(EQ15) $(A \Rightarrow B) \lor (D \Rightarrow (D \& (C \Rightarrow ((B \Rightarrow A) \& C))))$
Prelinear $\text{EQ}_\triangle$-logic

Language
The language of basic EQ-logic extended by unary connective $\Delta$, binary connective $\lor$ and logical constant $\bot$.

Negation $\neg A := A \equiv \bot$

Axioms (EQ1)–(EQ11) and

$(((A \land B) \lor C) \equiv D) \& (F \equiv C)) \& (E \equiv A) \Rightarrow (D \equiv (F \lor (B \land E)))$

(EQ12) $(A \lor B) \lor C \equiv A \lor (B \lor C)$

(EQ13) $A \lor (A \land B) \equiv A$

(EQ14) $(A \land \bot) \equiv \bot$

(EQ15) $(A \Rightarrow B) \lor (D \Rightarrow (D \& (C \Rightarrow ((B \Rightarrow A) \& C))))$
Prelinear $EQ_{\Delta}$-logic

Axioms (continued)

1. (EQΔ1) $\Delta A \Rightarrow \Delta\Delta A$
2. (EQΔ2) $\Delta(A \equiv B) \Rightarrow (\Delta A \equiv \Delta B)$
3. (EQΔ3) $\Delta(A \land B) \equiv (\Delta A \land \Delta B)$
4. (EQΔ4) $\Delta A \equiv (\Delta A \& \Delta A)$
5. (EQΔ5) $\Delta(A \lor B) \Rightarrow (\Delta A \lor \Delta B)$
6. (EQΔ6) $\Delta A \lor \neg\Delta A$
7. (EQΔ7) $\Delta(A \equiv B) \Rightarrow ((A \& C) \equiv (B \& C))$
8. (EQΔ8) $\Delta(A \equiv B) \Rightarrow ((C \& A) \equiv (C \& B))$
Prelinear $\text{EQ}_\Delta$-logic

Inference rules
- Equanimity rule
- Leibniz rule
- Necessitation rule

From $A$ infer $\Delta A$

Semantics
An $\ell \text{EQ}_\Delta$-algebras in which (1) is satisfied.
First-order EQ-logic with equality

Propositional EQ-logics

Prelinear $EQ_\Delta$-logic

Prelinear $EQ_\Delta$-logic

Inference rules

- Equanimity rule
- Leibniz rule
- Necessitation rule

From $A$ infer $\Delta A$

Semantics

An $\ell EQ_\Delta$-algebras in which (1) is satisfied.
Prelinear $\text{EQ}_{\Delta}$-logic

Theorem (Completeness)

For every formula $A \in F_J$ and every theory $T$ the following is equivalent:

(a) $T \vdash A$

(b) $e(A) = 1$ for every truth evaluation $e : F_J \rightarrow E$ and every linearly ordered, $\ell\text{EQ}_{\Delta}$-algebra $\mathcal{E}$.

(c) $e(A) = 1$ for every truth evaluation $e : F_J \rightarrow E$ and every $\ell\text{EQ}_{\Delta}$-algebra $\mathcal{E}$ satisfying (1).
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Fuzzy equality

Definition

Let $\mathcal{E}$ be a noncommutative EQ-algebra with the support $E$ and $M$ be a set. A fuzzy equality $\simeq$ on $M$ is a binary fuzzy relation on $M$, i.e. a function

$$\simeq : M \times M \rightarrow E$$

such that the following holds for all $m, m', m'' \in M$:

(i) $(m \simeq m) = 1$, \quad (reflexivity)

(ii) $(m \simeq m') = (m' \simeq m)$, \quad (symmetry)

(iii) $(m \simeq m') \otimes (m' \simeq m'') \leq (m \simeq m'')$ \quad (transitivity)

Function $f : M^n \rightarrow M$ is weakly extensional if

$$(m_1 \simeq m'_1), \ldots, (m_n \simeq m'_n) = 1$$ implies

$$(f(m_1, \ldots, m_n) \simeq f(m'_1, \ldots, m'_n)) = 1.$$
First-order EQ-logic with equality

Predicate EQ-logic

Syntax

Language

- Object variables $x, y, \ldots$.
- Set of object constants $\text{Const} = \{u, v, \ldots\}$.
- Set of n-ary functional symbols $\text{Func} = \{f, g, \ldots\}$.
- Non-empty set of n-ary predicate symbols $\text{Pred} = \{P, Q, \ldots\}$.
- Binary connectives $\wedge, \lor, \&$, $\equiv$ and unary connective $\Delta$.
- Binary symbol $\equiv$ for fuzzy equality between objects.
- Logical (truth) constants $\top$ (true) and $\bot$ (false).
- Quantifiers $\forall, \exists$.
- Auxiliary symbols: brackets.
First-order EQ-logic with equality

Predicate EQ-logic

Syntax

Terms

Object variables and object constants are terms.

Formulas

- If $P$ is an $n$-ary predicate symbol and $t_1, \ldots, t_n$ are terms then $P(t_1, \ldots, t_n)$ and $t_1 \equiv t_2$ are atomic formulas.
- Logical constants $\top$ and $\bot$ are formulas.
- If $A, B$ are formulas then $A \land B$, $A \lor B$, $A \& B$, $A \equiv B$, $\Delta A$ are formulas.
- If $A$ is formula and $x$ is an object variable then $(\forall x)A$, $(\exists x)A$ are formulas.
Philosophy is the study of the nature of existence, knowledge and the universe. It is a way of thinking about the world and our place in it. The study of philosophy can help us to understand our own beliefs and values, and to develop critical thinking skills.
Interpretation of terms and formulas

\( \nu \) — assignment of elements from \( M \) to variables

\[ M^\mathcal{E}_\nu(x) = \nu(x), \quad M^\mathcal{E}_\nu(u) = m_u, \]
\[ M^\mathcal{E}_\nu(f(t_1, \ldots, t_n)) = f_M(M^\mathcal{E}_\nu(t_1), \ldots, M^\mathcal{E}_\nu(t_n)) \]

\[ M^\mathcal{E}_\nu(P(t_1, \ldots, t_n)) = r_P(M^\mathcal{E}_\nu(t_1), \ldots, M^\mathcal{E}_\nu(t_n)), \]
\[ M^\mathcal{E}_\nu(t_1 \equiv t_2) = M^\mathcal{E}_\nu(t_1) \equiv M^\mathcal{E}_\nu(t_2), \]
\[ M^\mathcal{E}_\nu(A \land B) = M^\mathcal{E}_\nu(A) \land M^\mathcal{E}_\nu(B), \]
\[ M^\mathcal{E}_\nu(A \lor B) = M^\mathcal{E}_\nu(A) \lor M^\mathcal{E}_\nu(B), \]
\[ M^\mathcal{E}_\nu(A \& B) = M^\mathcal{E}_\nu(A) \otimes M^\mathcal{E}_\nu(B), \]
\[ M^\mathcal{E}_\nu(A \equiv B) = M^\mathcal{E}_\nu(A) \sim M^\mathcal{E}_\nu(B), \]
\[ M^\mathcal{E}_\nu(\Delta A) = \Delta M^\mathcal{E}_\nu(A), \quad M^\mathcal{E}_\nu(\top) = 1, \quad M^\mathcal{E}_\nu(\bot) = 0, \]
\[ M^\mathcal{E}_\nu(\forall x)A = \inf\{ M^\mathcal{E}_{\nu'}(A) \mid \nu' = \nu \setminus x \}, \]
\[ M^\mathcal{E}_\nu(\exists x)A = \sup\{ M^\mathcal{E}_{\nu'}(A) \mid \nu' = \nu \setminus x \}. \]
Logical Axioms

(EQ1)–(EQ15), (EQΔ1-EQΔ8) plus

(EQ∀1) \((\forall x)A(x) \Rightarrow A(t)\) (\(t\) substitutable for \(x\) in \(A(x)\)),

(EQ∃1) \(A(t) \Rightarrow (\exists x)A(x)\) (\(t\) substitutable for \(x\) in \(A(x)\)),

(EQ∀2) \(\Delta(\forall x)(A \Rightarrow B) \Rightarrow (A \Rightarrow (\forall x)B)\) (\(x\) not free in \(A\)),

(EQ∃2) \((\forall x)(A \Rightarrow B) \Rightarrow ((\exists x)A \Rightarrow B)\) (\(x\) not free in \(B\)),

(EQ∀3) \((\forall x)(A \vee B) \Rightarrow ((\forall x)A \vee B)\) (\(x\) not free in \(B\)),

First-order EQ-logic with equality

Predicate EQ-logic
Logical Axioms (continued)

(EQE1) \( t \equiv t \),

(EQE2) \( (s \equiv t) \equiv (t \equiv s) \),

(EQE3) \( (r \equiv s) \& (s \equiv t) \Rightarrow (r \equiv t) \),

(EQE4) \( (s \equiv t) \& (r \equiv s) \Rightarrow (r \equiv t) \),

(EQE5) \( \Delta(t_1 \equiv s_1) \Rightarrow (\ldots \Rightarrow (\Delta(t_n \equiv s_n) \Rightarrow \ldots) \Rightarrow (f(t_1, \ldots, t_n) \equiv f(s_1, \ldots, s_n)) \ldots) \),

(EQE6) \( \Delta(t_1 \equiv s_1) \Rightarrow (\ldots \Rightarrow (\Delta(t_n \equiv s_n) \Rightarrow \ldots) \Rightarrow (P(t_1, \ldots, t_n) \equiv P(s_1, \ldots, s_n)) \ldots) \).
Inference Rules

- Equanimity rule
- Leibniz rule

\[ \text{From } A \equiv B \ \text{infer} \ C[p := A] \equiv C[p := B], \]
provided that \( p \) is not in the scope of a quantifier in \( C \).

- Necessitation rule
- Rule of Generalization

\[ \text{From } A \ \text{infer} \ (\forall x)A \]
Model

Definition
Structure $\mathcal{M}^\mathcal{E}$ is a model of a theory $T$ if $\mathcal{M}^\mathcal{E}_v(A) = 1$ holds for all axioms $A$ of $T$.

Theorem (Soundness)
If $T \vdash A$ then $\mathcal{M}^\mathcal{E}_v(A) = 1$ holds for every assignment $v$ and every model $\mathcal{M}^\mathcal{E}$ of $T$. 
Main properties

**Lemma**

(i) \(\vdash (\forall x)(A \Rightarrow B) \equiv (A \Rightarrow (\forall x)B), \ x \text{ not free in } A\)

(ii) \(\vdash (\forall x)(B \Rightarrow A) \equiv ((\exists x)B \Rightarrow A), \ x \text{ not free in } A\)

(iii) \(\vdash \Delta(\forall x)(A \Rightarrow B) \Rightarrow ((\forall x)A \Rightarrow (\forall x)B)\)

(iv) \(\vdash (\forall x)(A \Rightarrow B) \Rightarrow ((\exists x)A \Rightarrow (\exists x)B)\)

**Theorem (Deduction)**

For each theory \(T\), closed formulas \(A, B\) and arbitrary formula \(C\):

\[ T \cup \{A \equiv B\} \vdash C \iff T \vdash \Delta(A \equiv B) \Rightarrow C. \]
Completeness

Definition

(i) Theory $T$ is consistent if there is a formula $A$ unprovable in $T$.

(ii) $T$ is linear (complete) if for every two formulas $A, B$, $T \vdash A \implies B$ or $T \vdash B \implies A$.

(iii) $T$ is extensionally complete if for every closed formula $(\forall x)(A(x) \equiv B(x))$, $T \not\vdash (\forall x)(A(x) \equiv B(x))$ there is a constant $u$ such that $T \not\vdash (A_x[u] \equiv B_x[u])$
Completeness

Theorem

Every consistent theory $T$ can be extended to a maximally consistent linear theory.

Theorem

Every consistent theory $T$ can be extended to an extensionally complete consistent theory $T$. 
Completeness

Theorem (Completeness)

(a) A theory $T$ is consistent iff it has a safe model $M$.

(b) $T \vdash A$ iff $T \models A$
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Conclusion

- Formal system of predicate EQ-logic with fuzzy equality in which equivalence is the main connective.
- $\Delta$-connective (deduction theorem) is indispensable for development of the first-order EQ-logic.
- Truth structure ($\mathcal{L}EQ_\Delta$-algebra) must be linearly ordered.
Thank you for your attention.