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A framework for fuzzy models of multiple-criteria evaluation

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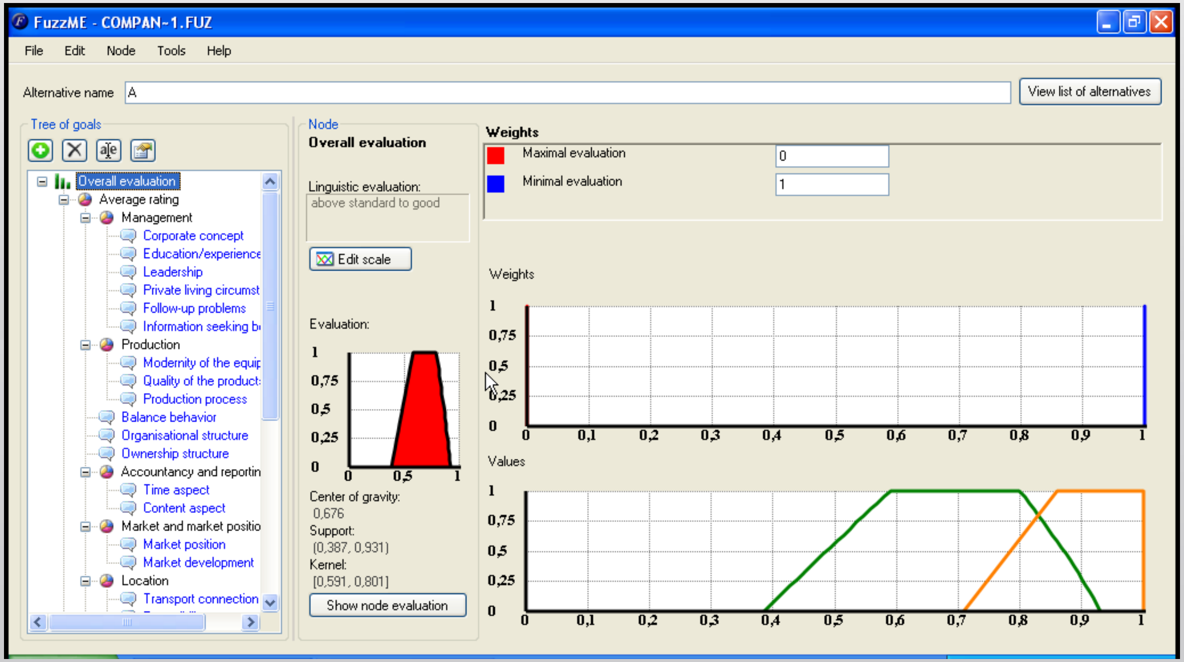
The FuzzME software package

FuzzME = Fuzzy models of Multiple-criteria Evaluation (2010)

- Successor of the Nefrit software package (1999)



The multiple-criteria evaluation problem

The problem of study is to construct a mathematical model for evaluating alternatives with respect to a given goal, the fulfillment of which can be measured by a set of *m* criteria. Moreover:

1. The set of alternatives is not supposed to be known in advance; the evaluation procedure must be applicable to individual incoming alternatives.
2. The complex case of multiple-criteria evaluation is considered:
   - the number of criteria is large,
   - the structure of evaluator's preferences on the criteria space is complex.
3. The model must be able to process expertly-defined data and use expert knowledge related to the evaluation process. Outputs from the model must be as much intelligible as possible.

Type of evaluation used

- As we not only compare alternatives within a pre-specified set but we need to assess alternatives entering into the system one by one, we **cannot work with evaluation of a relative type**.
- We must consider an **evaluation of absolute type with respect to a given goal**.
- An appropriate **crisp scale of evaluation** is the interval [0,1] with the following interpretation of its values:
  - 0 … the alternative **does not meet the goal at all**,
  - 1 ….the alternative **fully satisfies the goal**;
  - $\alpha \in (0,1)$…the **degree to which the goal has been fulfilled**.
- **The evaluation** of an alternative can be conceived of as **a membership degree to the fuzzy goal** (see also Bellman & Zadeh, 1970).

Type of evaluation used

In the evaluation models described further the evaluations are modeled by **fuzzy numbers** defined on the interval [0,1].

Comments:

- A fuzzy number U is said to be defined on [0,1] if $Supp\, U \subseteq [0,1]$.
- A set of fuzzy numbers defined on [0,1] - $\mathcal{F}_N([0,1])$
- Any fuzzy number U can be characterized by a pair of functions

$$\underline{u} : [0,1] \to \Re, \overline{u} : [0,1] \to \Re :$$

$$\left[\underline{u}(\alpha), \overline{u}(\alpha)\right] = U_\alpha \text{ for all } \alpha \in (0,1],$$

$$\left[\underline{u}(0), \overline{u}(0)\right] = Cl(Supp\, U).$$

Therefore, the fuzzy number U can also be written as:

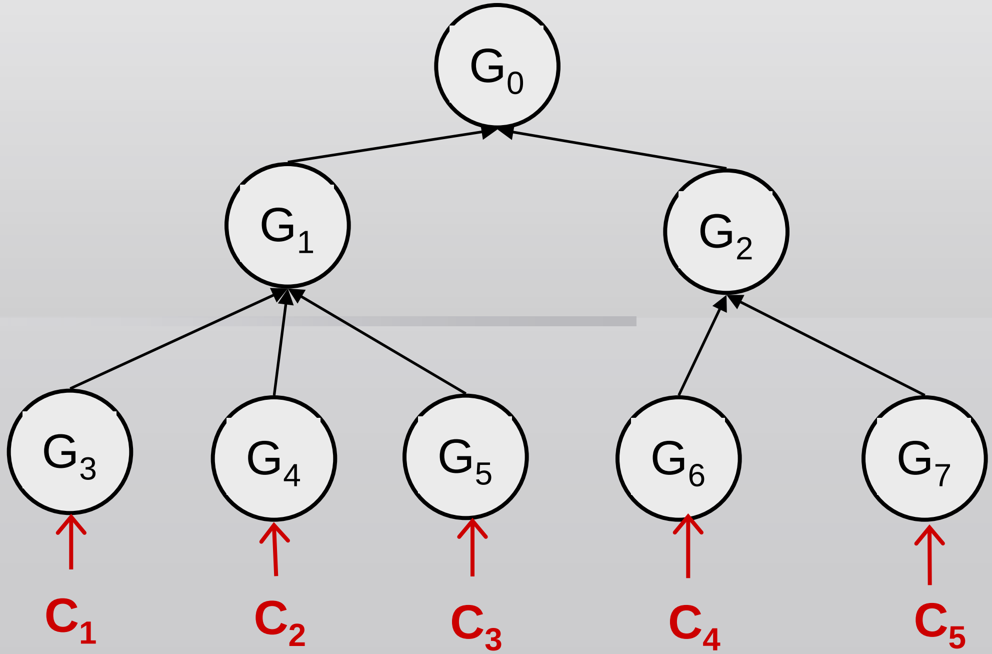
$$\boxed{U = \left\{\left[\underline{u}(\alpha), \overline{u}(\alpha)\right], \alpha \in [0,1]\right\}.}$$

Type of evaluation used

- These fuzzy evaluations on [0,1] express **uncertain degrees of fulfillment of the given goal** by respective alternatives.
- **Goals** correspond with **type-2 fuzzy sets** of alternatives.
- The used **aggregation methods preserve the type of evaluation**.
- Fuzzy evaluations expressing uncertain degrees of goals fulfillment will be implemented in the presented models on all levels of evaluation.

The basic structure of evaluation model

- The evaluation structure is expressed by a **goals tree**.
- Partial goals at the ends of the branches are connected with quantitative or qualitative criteria.

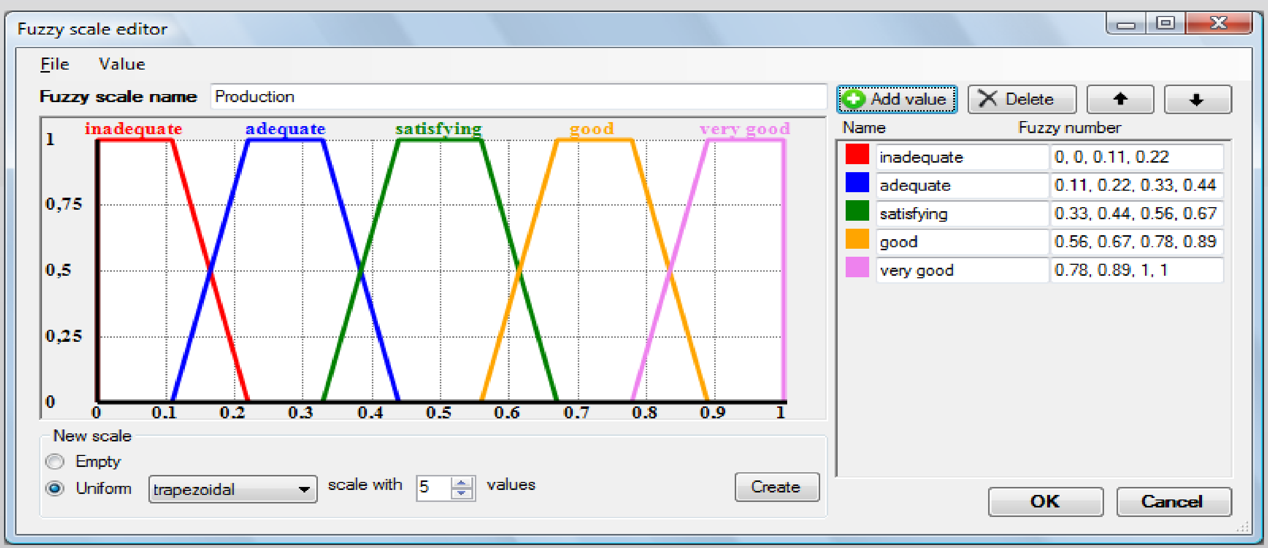


Process of evaluation

1. Partial fuzzy evaluations with respect to criteria
2. Consecutive aggregation of partial evaluations by means of:
   - fuzzified aggregation operators
   - fuzzy expert systems
3. The overall fuzzy evaluation

Evaluations according to qualitative criteria

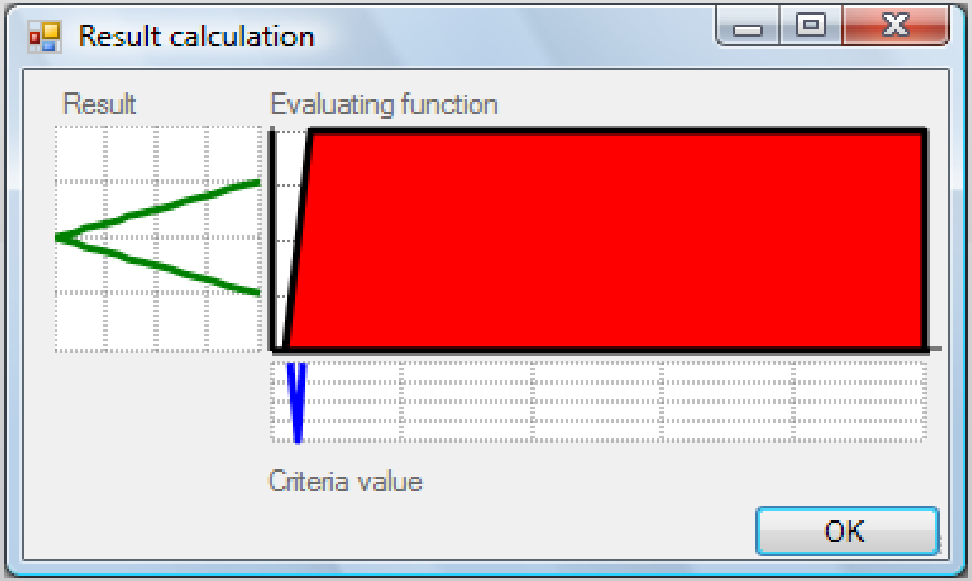
- Alternatives are evaluated verbally, by means of values of the linguistic variables of special types:
  - linguistic scales, e.g. “good”
  - extended linguistic scales, e.g. “good to very good”
  - linguistic scales with intermediate values, “between good and very good”



Evaluations according to quantitative criteria

- Evaluations are calculated:
  - from the measured value of the criterion (crisp or fuzzy)
  - by means of the expertly defined evaluating function, membership function of the corresponding partial goal.

criterion: expected profitability of a project



Aggregation of the partial evaluations

- The partial fuzzy evaluations are consecutively aggregated according to the structure of the goals tree
- Supported aggregation methods:
  - **FuzzyWA,**
  - **FuzzyOWA,**
  - **fuzzified WOWA,**
  - **fuzzified Choquet integral,**
  - **fuzzy expert system.**

Fuzzified aggregation operators - normalized fuzzy weights

- **Definition**

  Fuzzy numbers $V_1, \ldots, V_m$ defined on [0,1] are called normalized fuzzy weights if for any $i \in \{1, 2, \ldots m\}$ and any $\alpha \in (0,1]$ the following holds:

  For any $v_i \in V_{i\alpha}$ there exist $v_j \in V_{j\alpha}$, $j=1,\ldots,m$, $j \neq i$, such that

$$v_i + \sum_{\substack{j=1 \\ j \neq i}}^{m} v_j = 1.$$

Fuzzified aggregation operators
- Fuzzy Weighted Average

- **Definition**

The FuzzyWA of the partial fuzzy evaluations, i.e., of fuzzy numbers $U_1, \ldots, U_m$ defined on $[0,1]$, with the normalized fuzzy weights $V_1, \ldots, V_m$, is a fuzzy number U on $[0,1]$ whose membership function is defined for any $u \in [0,1]$ as follows

$$U(u) = \max\{\min\{V_1(v_1), \ldots, V_m(v_m), U_1(u_1), \ldots, U_m(u_m)\} \mid \sum_{i=1}^{m} v_i \cdot u_i = u, \boxed{\sum_{i=1}^{m} v_i = 1}, v_i, u_i \in [0,1], i = 1, \ldots, m\}.$$

- An effective algorithm was found for its calculation (Pavlačka).

Fuzzy Weighted Average - algorithm

1. Let σ and τ be permutations of the set of indices $\{1, ..., m\}$ such that $\underline{u}_{\sigma(1)}(\alpha) \leq ... \leq \underline{u}_{\sigma(m)}(\alpha)$ and $\overline{u}_{\tau(1)}(\alpha) \geq ... \geq \overline{u}_{\tau(m)}(\alpha)$.
2. Let for $k \in \{1, ..., m\}$ the values $v_k^L(\alpha)$ and $v_k^R(\alpha)$ be given as

$$v_k^L(\alpha) = 1 - \sum_{i=1}^{k-1} \overline{v}_{\sigma(i)}(\alpha) - \sum_{i=k+1}^{m} \underline{v}_{\sigma(i)}(\alpha),$$

$$v_k^R(\alpha) = 1 - \sum_{i=1}^{k-1} \overline{v}_{\tau(i)}(\alpha) - \sum_{i=k+1}^{m} \underline{v}_{\tau(i)}(\alpha).$$

3. Let k^* and k^{**} denote such indices that the following holds:

$$\underline{v}_{\sigma(k^*)}(\alpha) \leq v_{k^*}^L(\alpha) \leq \overline{v}_{\sigma(k^*)}(\alpha), \quad \underline{v}_{\tau(k^{**})}(\alpha) \leq v_{k^{**}}^R(\alpha) \leq \overline{v}_{\tau(k^{**})}(\alpha).$$

4. Then

$$\underline{u}(\alpha) = \sum_{i=1}^{k^*-1} \overline{v}_{\sigma(i)}(\alpha) \cdot \underline{u}_{\sigma(i)}(\alpha) + v_{k^*}^L(\alpha) \cdot \underline{u}_{\sigma(k^*)}(\alpha) + \sum_{i=k^*+1}^{m} \underline{v}_{\sigma(i)}(\alpha) \cdot \underline{u}_{\sigma(i)}(\alpha),$$

$$\overline{u}(\alpha) = \sum_{i=1}^{k^{**}-1} \overline{v}_{\tau(i)}(\alpha) \cdot \overline{u}_{\tau(i)}(\alpha) + v_{k^{**}}^R(\alpha) \cdot \overline{u}_{\tau(k^{**})}(\alpha) + \sum_{i=k^{**}+1}^{m} \underline{v}_{\tau(i)}(\alpha) \cdot \overline{u}_{\tau(i)}(\alpha).$$

Fuzzified aggregation operators
- Fuzzy Weighted Average

- **Example**

As an example of FuzzyWA application, let us consider academic staff performance evaluation in three areas – research, teaching, and administration. Each area has a different importance. First we set the crisp weights $v_{research} = 0.4$, $v_{teaching} = 0.4$ and $v_{administration} = 0.2$. Because we do not know the importances precisely, we add some uncertainty. The normalized fuzzy weights can be, e.g., $V_{research} = (0.3, 0.4, 0.5)$, $V_{teaching} = (0.3, 0.4, 0.5)$ and $V_{administration} = (0.1, 0.2, 0.3)$.

Fuzzified aggregation operators
- Fuzzy Ordered Weighted Average

- **Definition**

The FuzzyOWA of the partial fuzzy evaluations, i.e., of fuzzy numbers $U_1, ..., U_m$ defined on $[0,1]$, with normalized fuzzy weights $V_1, \ldots, V_m$, is a fuzzy number U on $[0,1]$ whose membership function is defined for any $u \in [0,1]$ as follows

$$U(u) = \max\{\min\{V_1(v_1), ..., V_m(v_m), U_1(u_1), ..., U_m(u_m)\} \mid \sum_{i=1}^{m} v_i \cdot u_{\phi(i)} = u, \boxed{\sum_{i=1}^{m} v_i = 1,} \ v_i, u_i \in [0,1], i = 1, ..., m\}, \quad (4)$$

where ϕ denotes such a permutation of the set of indices $\{1, ..., m\}$ that $u_{\phi(1)} \geq u_{\phi(2)} \geq ... \geq u_{\phi(m)}$.

- A similar algorithm as for FuzzyWA was developed (Bebčáková).

Fuzzy Ordered Weighted Average - Algorithm

1. Let σ and τ be permutations of the set of indices $\{1, ..., m\}$ such that $\underline{u}_{\sigma(1)}(\alpha) \leq ... \leq \underline{u}_{\sigma(m)}(\alpha)$ and $\overline{u}_{\tau(1)}(\alpha) \geq ... \geq \overline{u}_{\tau(m)}(\alpha)$.
2. Let for $k \in \{1, ..., m\}$ the values $v_k^L(\alpha)$ and $v_k^R(\alpha)$ be given as

$$v_k^L(\alpha) = 1 - \sum_{i=1}^{k-1} \underline{v}_i(\alpha) - \sum_{i=k+1}^{m} \overline{v}_i(\alpha),$$

$$v_k^R(\alpha) = 1 - \sum_{i=1}^{k-1} \overline{v}_i(\alpha) - \sum_{i=k+1}^{m} \underline{v}_i(\alpha).$$

3. Let k^* and k^{**} denote such indices that the following holds:

$$\underline{v}_{k^*}(\alpha) \leq v_{k^*}^L(\alpha) \leq \overline{v}_{k^*}(\alpha), \quad \underline{v}_{k^{**}}(\alpha) \leq v_{k^{**}}^R(\alpha) \leq \overline{v}_{k^{**}}(\alpha).$$

4. Then

$$\underline{u}(\alpha) = \sum_{i=1}^{m-k^*} \overline{v}_{m-i+1}(\alpha) \cdot \underline{u}_{\sigma(i)}(\alpha) + v_{k^*}^L(\alpha) \cdot \underline{u}_{\sigma(m-k^*+1)}(\alpha) + \sum_{i=m-k^*+2}^{m} \underline{v}_{m-i+1}(\alpha) \cdot \underline{u}_{\sigma(i)}(\alpha),$$

$$\overline{u}(\alpha) = \sum_{i=1}^{k^{**}-1} \overline{v}_i(\alpha) \cdot \overline{u}_{\tau(i)}(\alpha) + v_{k^{**}}^R(\alpha) \cdot \overline{u}_{\tau(k^{**})}(\alpha) + \sum_{i=k^{**}+1}^{m} \underline{v}_i(\alpha) \cdot \overline{u}_{\tau(i)}(\alpha).$$

Fuzzified aggregation operators - Fuzzy Ordered Weighted Average

- **Example 1**

As an example, let us consider an HR agent wishing to employ new workers whose partial fuzzy evaluations are known at the job interview. In his/her view, only those candidates who are not significantly bad according to all criteria can be hired. Then the weight of the minimum partial evaluation of each worker equals 1, and the weights of all other partial evaluations equal 0. The aggregated fuzzy evaluations then represent the guaranteed fuzzy degrees of fulfillment of all the partial goals (the FuzzyMin Method).

Fuzzified aggregation operators
- Fuzzy Ordered Weighted Average

- **Example 2**

Another example of using the FuzzyOWA operator is the evaluation of workers by their colleagues. Because in every team there are friends and foes, we will ignore the best and the worst partial evaluation. The other partial evaluations have the same importance so their weights will be uniform. If we have, for example, the evaluation from five people, then the weights for OWA will be $(0, \frac{1}{3}, \frac{1}{3}, \frac{1}{3}, 0)$; in the fuzzy case, a fuzzy number, e.g. $(\frac{1}{4}, \frac{1}{3}, \frac{1}{2})$ would be used instead of the real weight $\frac{1}{3}$.

Fuzzified aggregation operators - Fuzzified WOWA

- **Definition**

The fuzzified WOWA operator, considered in this paper, is able to aggregate the fuzzy partial evaluations $U_i = \{[\underline{u}_i(\alpha), \overline{u}_i(\alpha)], \alpha \in [0,1]\}$, $i = 1, ..., m$. However, the weights $w_1, w_2, ..., w_m$ and $p_1, p_2, ..., p_m$ must be crisp. The result of the aggregation is a fuzzy number $U = \{[\underline{u}(\alpha), \overline{u}(\alpha)], \alpha \in [0,1]\}$ defined for any $\alpha \in [0,1]$ as follows

$$\underline{u}(\alpha) = \sum_{i=1}^{m} \omega_i^L \cdot \underline{u}_{\sigma(i)}(\alpha), \tag{5}$$

$$\overline{u}(\alpha) = \sum_{i=1}^{m} \omega_i^R \cdot \overline{u}_{\chi(i)}(\alpha), \tag{6}$$

Fuzzified aggregation operators
- Fuzzified WOWA

where σ, and χ are permutations of the set of indices $\{1, ..., m\}$ such that $\underline{u}_{\sigma(1)}(\alpha) \geq ... \geq \underline{u}_{\sigma(m)}(\alpha)$ and $\overline{u}_{\chi(1)}(\alpha) \geq ... \geq \overline{u}_{\chi(m)}(\alpha)$. The weights ω_i^L and ω_i^R, $i = 1, .., m$, are defined for the given α as

$$\omega_i^L = z(\sum_{j \leq i} w_{\sigma(j)}) - z(\sum_{j<i} w_{\sigma(j)}), \tag{7}$$

$$\omega_i^R = z(\sum_{j \leq i} w_{\chi(j)}) - z(\sum_{j<i} w_{\chi(j)}), \tag{8}$$

where z is a nondecreasing piece-wise linear function interpolating the following points

$$\{(0,0)\} \cup \{(i/m, \sum_{j \leq i} p_j)\}_{i=1,...,m}. \tag{9}$$

Fuzzified aggregation operators
- Fuzzified WOWA

- **Example**

In the first example, concerning an application of weighted average, we evaluated academic staff. We used the following weights for the different partial goals: $w_{research} = 0.4$, $w_{teaching} = 0.4$, and $w_{administration} = 0.2$. Now we will extend the example. We give the academic staff members some freedom to choose in which of these three areas they will get involved the most. For the results in the area where the subject performs best, we set the weight $p_1 = 0.5$. The results from the second best area will have weight $p_2 = 0.3$. The last weight, corresponding to the area in which the subject performs poorest, will be $p_3 = 0.2$. The evaluations of the academic staff members in each area can be modeled by fuzzy numbers on $[0, 1]$ (expressing the meanings of linguistic evaluations).

Fuzzified aggregation operators - Fuzzified WOWA

- **Example**

Let us consider an academic staff member who excels in research, but has only average evaluation in the area of teaching and poor evaluation in the area of administration, then his/her evaluation in the mentioned areas will be weighted by the composite weights $\omega_{research} = 0.56$, $\omega_{teaching} = 0.32$, and $\omega_{administration} = 0.12$. On the other hand, an academic staff member who teaches in the first place, engages extensively in administration, but has only a few research results, will have these partial evaluations weighted by the composite weights $\omega_{research} = 0.18$, $\omega_{teaching} = 0.56$, and $\omega_{administration} = 0.26$.

Fuzzified aggregation operators - Choquet integral

- **Definition**

  A **fuzzy measure** on a finite nonempty set G, G = {G_1, G_2,,…, G_m} is a set function $\mu : \wp(G) \to [0,1]$ satisfying the following axioms:

  $$\mu(\emptyset) = 0, \quad \mu(G) = 1;$$
  $$C \subseteq D \text{ implies } \mu(C) \le \mu(D), \text{ for any } C, D \in \wp(G)$$

- A fuzzy measure (a capacity) is a generalization of a clasic normalized measure, where aditivity is replaced by **monotonicity**.

- In multiple criteria evaluation models a fuzzy measure (a capacity) describes relations of **redundancy** or **compatibility** that are present among the partial goals.

Fuzzified aggregation operators - Choquet integral

- In case of redundancy, partial goals are overlapping – they have something in common. Therefore, the significance of this set of overlapping goals is lower than the sum of weights of individual goals. Weighted average cannot be used for aggregation of partial evaluations because the evaluation of the overlapping part would be included several times.
- The opposite type of interaction is complementarity of partial goals. Fulfilling of all such partial goals brings some “additional value”. The total significance of the considered group of partial goals is then greater than the sum of significances of the individual goals. Again, the weighted average is not suitable for this case because this “additional value” would not be incorporated at all.

Fuzzified aggregation operators - Choquet integral

- **Example:** **redundancy - partial goals are overlapping**

  We want to evaluate high school students' aptitude for study of Science. The evaluation will be based on the students' test results in *Mathematics*, *Physics*, and *Chemistry*.

  The fuzzy measure of the partial goals will be:
  μ(*Mathematics*)=0.5, μ(*Physics*) = 0.4 and μ(*Chemistry*) = 0.3.

  Students who are good at Math are usually also good at Physics. The reason is that these two subjects have a lot in common.

  Therefore, we set the fuzzy measures:
  μ(*Mathematics*, *Physics*) = 0.7< μ(*Math*) + μ(*Physics*)=0.9.

  Similarly,
  μ(*Mathematics*, *Chemistry*)=0.6 and μ(*Physics*, *Chemistry*)=0.6.

  Naturally μ(*Math*, *Physics*, *Chemistry*) = 1, and $\mu(\emptyset) = 0$.

Fuzzified aggregation operators - Choquet integral

- **Example: complementarity**

  We would like to evaluate career perspective of young mathematicians according to three criteria – *Mathematical Ability*, *English Proficiency* and *Communication Skills*. The knowledge of Math is the most important for them but without the other skills they will not be able to publish and present their results, which is a necessity in science. The significances of sets of partial goals can be expressed by a fuzzy measure, say:

  $\mu(\emptyset) = 0$,

  $\mu(\mathit{Math}) = 0.7$, $\mu(\mathit{English}) = 0.1$, $\mu(\mathit{Communication}) = 0.05$,

  $\mu(\mathit{Math}, \mathit{English}) = 0.85$, $\mu(\mathit{Math}, \mathit{Communication}) = 0.8$,

  $\mu(\mathit{English}, \mathit{Communication}) = 0.2$,

  $\mu(\mathit{Math}, \mathit{English}, \mathit{Communication}) = 1$.

Fuzzified aggregation operators - Choquet integral

- **Definition**

To define the Choquet integral, the following notation will be used. For any m-tuple of real numbers $(u_1, ..., u_m)$, ρ will denote such a permutation of the set of indices $\{1, ..., m\}$ that $u_{\rho(1)} \leq u_{\rho(2)} \leq ... \leq u_{\rho(m)}$. Moreover, let us denote $B_{\rho(i)} = \{G_{\rho(i)}, ..., G_{\rho(m)}\}$. By definition, we will set $B_{\rho(m+1)} = \emptyset$.

In the crisp case, the Choquet integral is used for aggregating partial evaluations in the following way. Let real numbers $u_1, ..., u_m$ be partial evaluations with respect to the goals $G_1, ..., G_m$. Let the importance of the partial-goal sets be defined by a fuzzy measure μ on G. Then the crisp Choquet integral is calculated as follows

$$(C)\int_G f d\mu = \sum_{i=1}^{m} f(G_{\rho(i)}) \cdot \left[\mu(B_{\rho(i)}) - \mu(B_{\rho(i+1)})\right], \tag{10}$$

where $f(G_i) = u_i$.

Fuzzified aggregation operators - fuzzified Choquet integral

- FNV-fuzzy measure is used – the importance of each subset of partial goals is expressed by a fuzzy number.
- **Definition**

  A FNV-fuzzy measure on a finite nonempty set G, G = {G_1, G_2,,…, G_m} is a set function $\tilde{\mu} : \wp(G) \to \mathcal{F}_N\left(\left[0,1\right]\right)$ satisfying the following axioms:

  $\tilde{\mu}(\emptyset) = 0,\ \tilde{\mu}(G) = 1$

  $C \subseteq D$ implies $\tilde{\mu}(C) \leq \tilde{\mu}(D)$, for any $C, D \in \wp(G)$

Fuzzified aggregation operators - fuzzified Choquet integral

- **Definition**

$\tilde{f}(G_i) = U_i, i = 1,\ldots,m$

Analogically, the Choquet integral of a FNV-function $\tilde{f}$ with respect to the FNV-fuzzy measure $\tilde{\mu}$ is defined as a fuzzy number U with a membership function given for any $u \in [0,1]$ by

$$U(u) = \max\Big\{ \min\{U_1(u_1), \ldots, U_m(u_m), \tilde{\mu}(B_{\rho(1)})(\mu_1), \ldots, \tilde{\mu}(B_{\rho(m)})(\mu_m)\} \mid$$
$$u = (C)\int_G f\,d\mu, \text{ where } f : G \to [0,1] \text{ such that } f(G_i) = u_i, i = 1, \ldots, m,$$
$$\mu \text{ is a fuzzy measure on } G \text{ such that } \mu(B_{\rho(i)}) = \mu_i, i = 1, \ldots, m\Big\}.$$

Fuzzified aggregation operators - fuzzified Choquet integral - algorithm

Let us denote $U_i = \{[\underline{u}_i(\alpha), \overline{u}_i(\alpha)], \alpha \in [0,1]\}$, $i = 1, ..., m$. Then the fuzzy value U of the fuzzified Choquet integral, $U = \{[\underline{u}(\alpha), \overline{u}(\alpha)], \alpha \in [0,1]\}$, can be calculated for any $\alpha \in [0,1]$ as follows:

$$\underline{u}(\alpha) = (C)\int_G f_L \, d\mu_L, \tag{12}$$

where $f_L : G \to [0,1]$ is a function such that $f_L(G_i) = \underline{u}_i(\alpha)$, $i = 1, ..., m$, and $\mu_L : \wp(G) \to [0,1]$ is such a fuzzy measure that $\mu_L(C) = \underline{\tilde{\mu}}(C)(\alpha)$ for any $C \subseteq G$.

$$\overline{u}(\alpha) = (C)\int_G f_R \, d\mu_R, \tag{13}$$

where $f_R : G \to [0,1]$ is a function such that $f_R(G_i) = \overline{u}_i(\alpha)$, $i = 1, ..., m$, and $\mu_R : \wp(G) \to [0,1]$ is such a fuzzy measure that $\mu_R(C) = \overline{\tilde{\mu}}(C)(\alpha)$ for any $C \subseteq G$.

Aggregation by fuzzy expert systems

- It can be applied, even if the relationship between the partial evaluations and the total evaluation is very complicated. (Fuzzy approximation theorems)
- Evaluating function is defined linguistically by a fuzzy rule base.
- Inference algorithms available in the system:
  - **Mamdani inference**
  - **generalized Sugeno inference:**
    - **Sugeno – WA, Sugeno - WOWA**

Modernity of	Ownership	Market position	Market	Dependences	Risk rate
inadequate	-unknown-	-unknown-	-unknown-	-unknown-	High risk
inadequate	unrateable or un	little presence in	falling sector	very high	Very high risk
-unknown-	unrateable or un	little presence in	-unknown-	-unknown-	Very high risk
-unknown-	-unknown-	-unknown-	falling sector	very high	High risk
inadequate	-unknown-	little presence in	-unknown-	-unknown-	Very high risk
-unknown-	-unknown-	little presence in	falling sector	-unknown-	Very high risk

Fuzzy expert systems

The fuzzy-rule base models the relationship between the partial evaluations of lower level and the aggregated evaluation. All the rules are in the following form:

$$\text{If } \mathcal{E}_i \text{ is } \mathcal{U}_{i,1} \text{ and } \ldots \text{ and } \mathcal{E}_m \text{ is } \mathcal{U}_{i,m}, \text{ then } \mathcal{E} \text{ is } \mathcal{U}_i, \tag{14}$$

where for $i = 1, 2, \ldots, n$, $j = 1, 2, \ldots, m$:

- $(\mathcal{E}_j, \mathcal{T}(\mathcal{E}_j), [0,1], M_j, G_j)$ are linguistic scales representing partial evaluations,
- $\mathcal{U}_{ij} \in \mathcal{T}(\mathcal{E}_j)$ are their linguistic values and $U_{ij} = M_j(\mathcal{U}_{ij})$ are fuzzy numbers on $[0,1]$ representing their meanings,
- $(\mathcal{E}, \mathcal{T}(\mathcal{E}), [0,1], M, G)$ is a linguistic scale representing the overall evaluation,
- $\mathcal{U}_i \in \mathcal{T}(\mathcal{E})$ are its linguistic values and $U_i = M(\mathcal{U}_i)$ are fuzzy numbers on $[0,1]$ representing their meanings.

Mamdani inference algorithm

In case of Mamdani fuzzy inference (Mamdani and Assilian, 1975), calculation proceeds in three steps.

1. First, the degree h_i of correspondence between the given m-tuple of fuzzy values $(U'_1, U'_2, ..., U'_m)$ of partial evaluations and the mathematical meaning of the left-hand side of the i-th rule is calculated for any $i = 1, ..., n$ in the following way

$$h_i = \min\{hgt(U'_1 \cap U_{i,1}), ..., hgt(U'_m \cap U_{i,m})\}. \tag{15}$$

2. For each of the rules, the output fuzzy value U''_i, $i = 1, ..., n$, corresponding to the given input fuzzy values, is calculated as follows

$$\forall y \in [0, 1] : U''_i(y) = \min\{h_i, U_i(y)\}. \tag{16}$$

3. The final fuzzy evaluation of the alternative is given as a union of all the fuzzy evaluations that were calculated for the particular rules in the previous step, i.e.,

$$U = \bigcup_{i=1}^{n} U''_i. \tag{17}$$

Sugeno WA inference algorithm

The result of the Sugeno-WA inference is obtained as follows.

1. In its first step, the degrees of correspondence h_i, $i = 1, ...n$, are calculated in the same way as in the Mamdani fuzzy inference algorithm.
2. The resulting fuzzy evaluation U is then computed as a weighted average of the fuzzy evaluations U_i, $i = 1, ..., n$, which model mathematical meanings of linguistic evaluations on the right-hand sides of the rules, with the weights h_i. This is done by the formula

$$U = \frac{\sum_{i=1}^{n} h_i \cdot U_i}{\sum_{i=1}^{n} h_i}. \tag{18}$$

Sugeno WOWA inference algorithm

For more complex cases, Sugeno-WOWA inference can be used (Holeček and Talašová, 2010). This method requires, besides a fuzzy rule base, normalized weights $p_1, p_2, ..., p_k$. These normalized weights are assigned to individual values of the linguistic scale representing the output variable $\mathcal{E}$. By these weights, the expert can express his/her optimism or pessimism (a pessimist assigns larger weights to bad evaluations, while an optimist to good evaluations).

Overall fuzzy evaluation

- The final result of the consecutive aggregation
- Fuzzy number on [0,1], degree of the total goal satisfaction.
- The user obtains:
  - graphic representation
  - linguistic approximation
    (by means of a linguistic scale, extended linguistic scale, linguistic scale with intermediate values)
  - centre of gravity, measure of uncertainty

Application of FuzzME in banking

- **Soft-fact-rating problem of one of the Austrian banks**:
  - companies evaluation,
  - decision making about granting a credit,

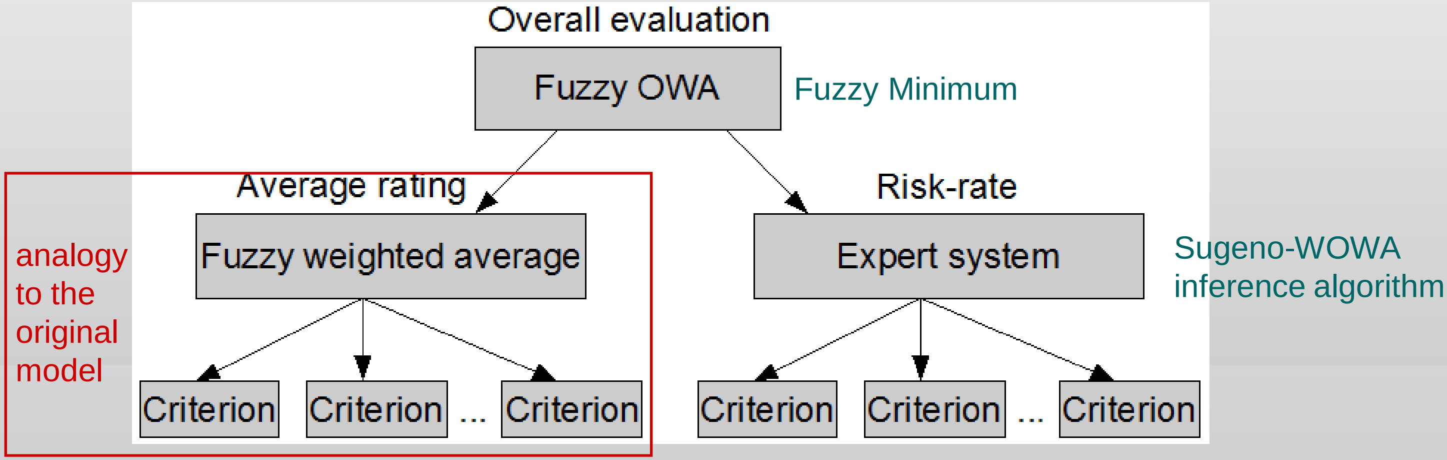
  solved in co-operation with TU Vienna

- Soft-fact-rating x hard-fact-rating

- **The original soft-fact-rating model of the bank:**
  - criteria - 27 qualitative criteria
  - partial evaluations - discrete numeric scales with linguistic descriptors
  - aggregation – standard weighted average

Application of FuzzME in banking

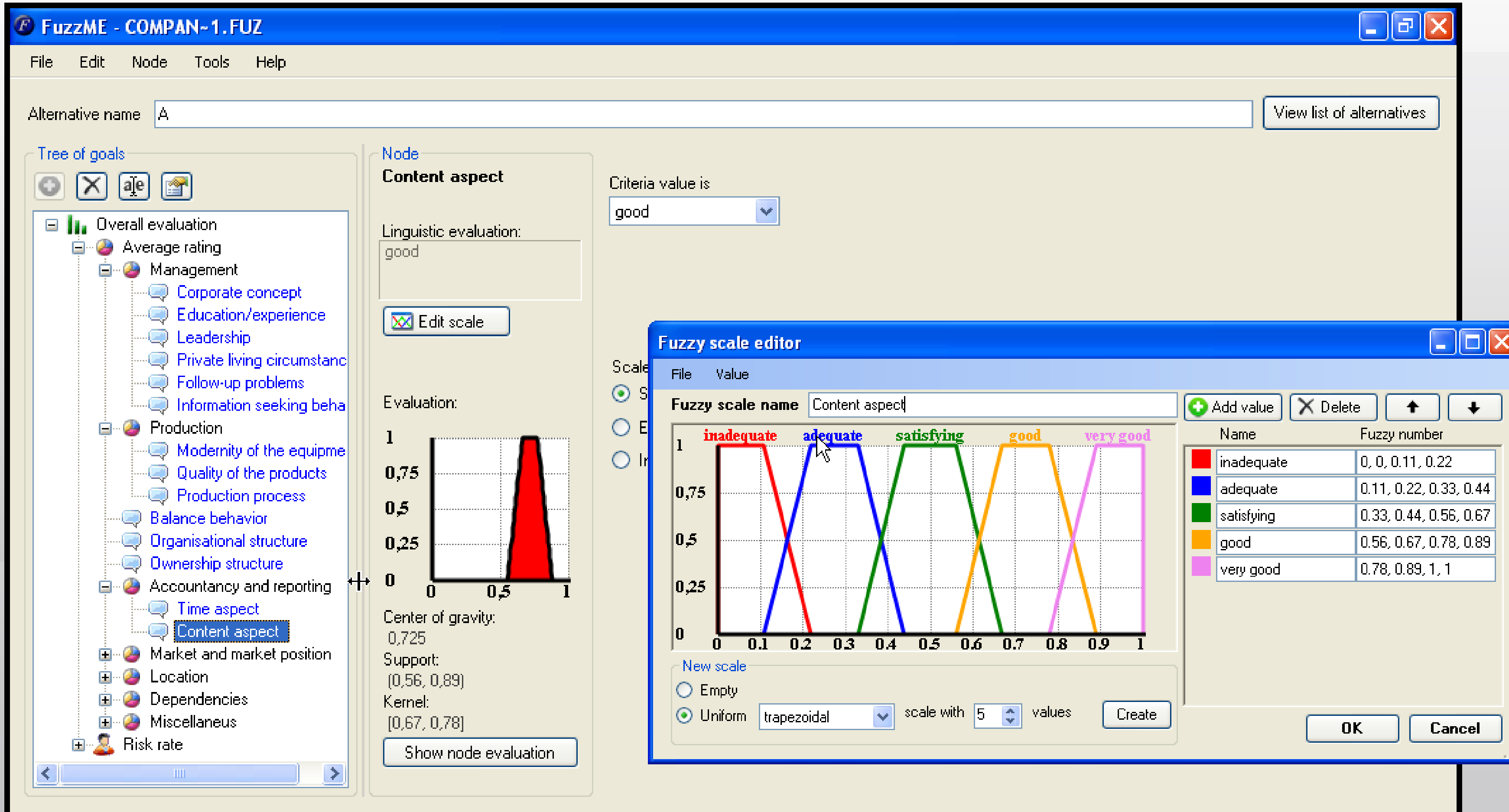
- **Proposed fuzzy model:**
  - partial evaluations - by linguistic fuzzy scales
  - aggregation:



  - the overall evaluation – linguistic approximation
    graphical representation, centre of gravity

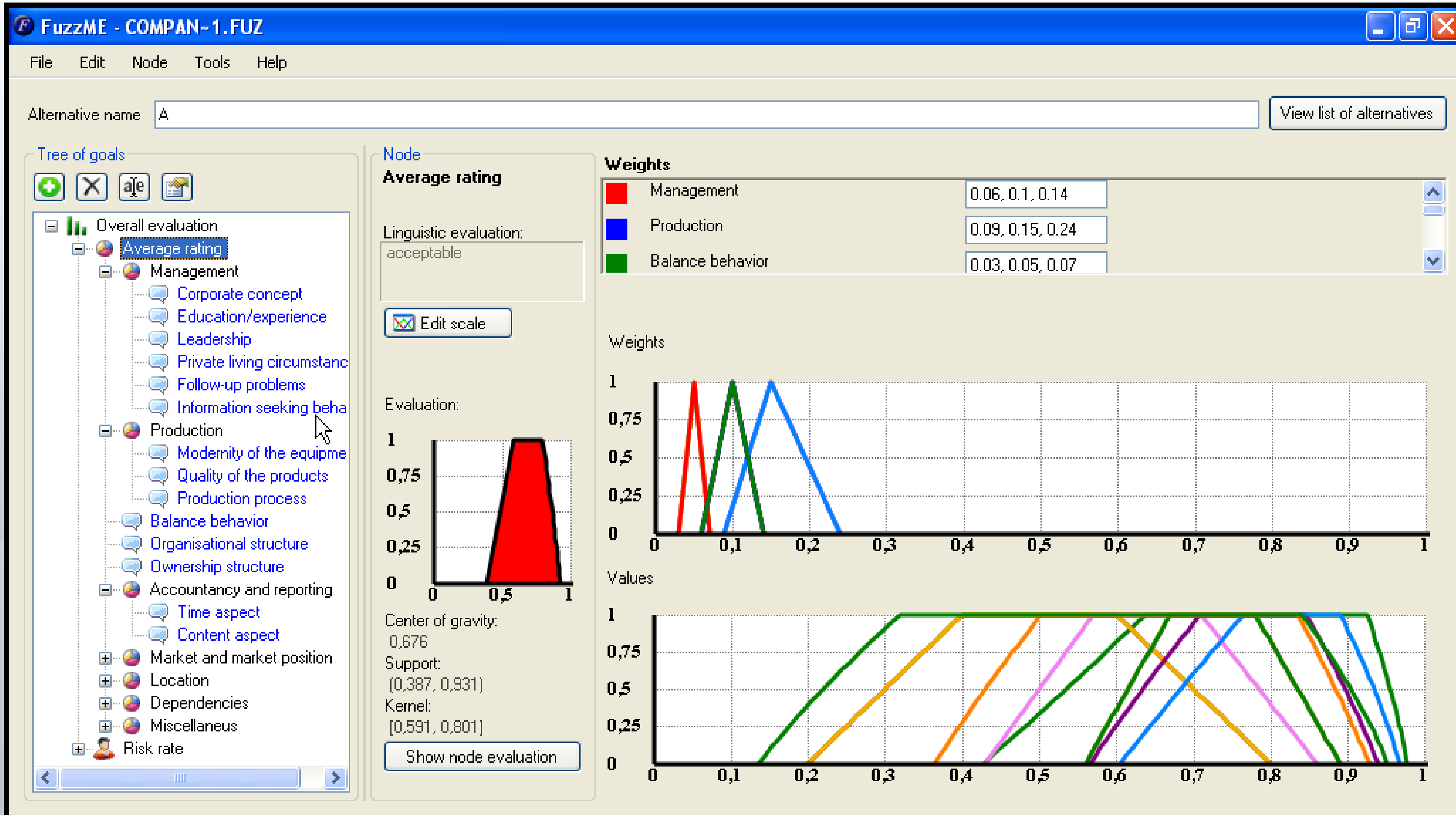
Application of FuzzME in banking

Fuzzy evaluation with respect to a qualitative criterion



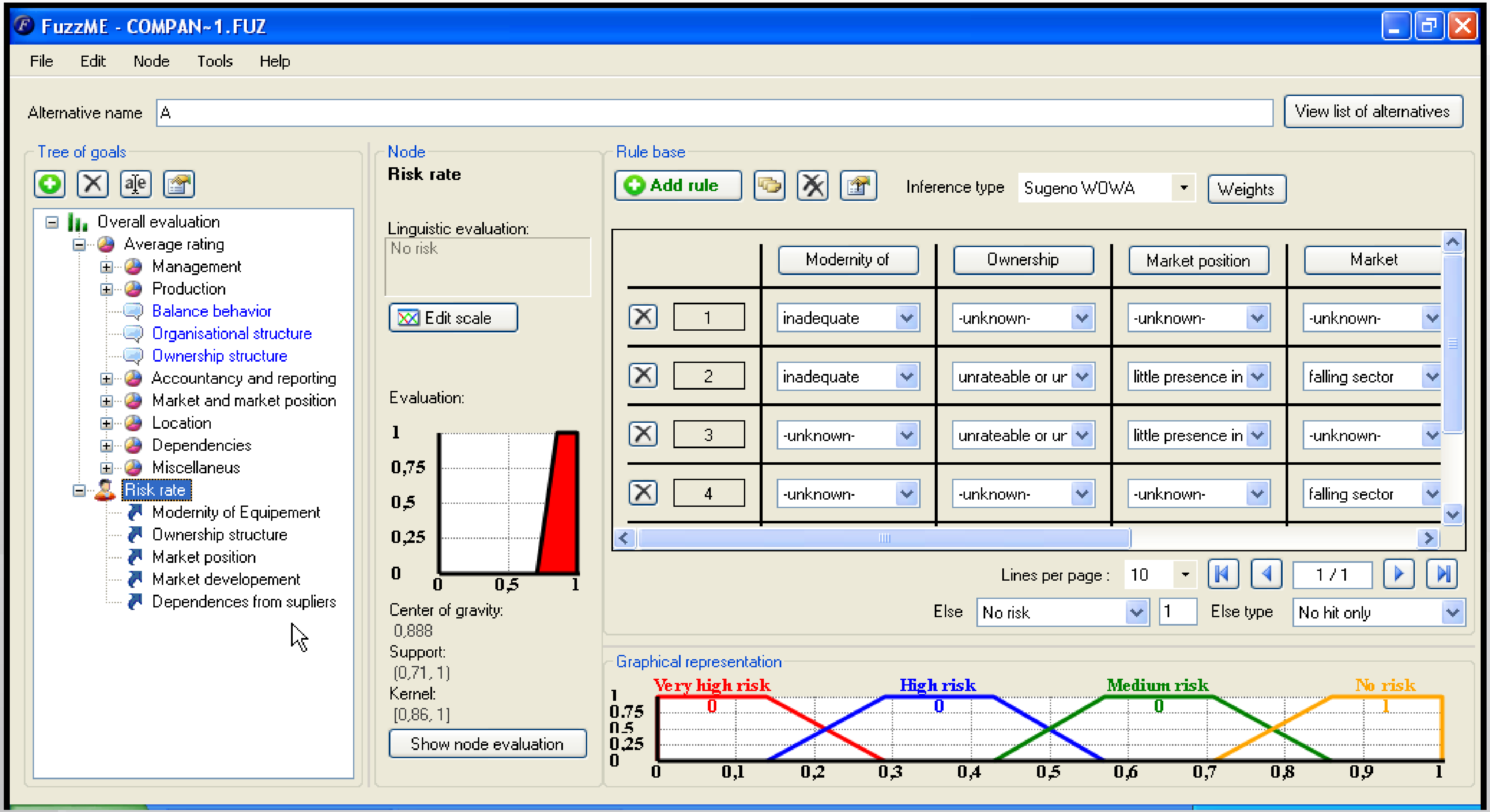
Application of FuzzME in banking

Fyzzy weighted average aggregation: *Average Rating*



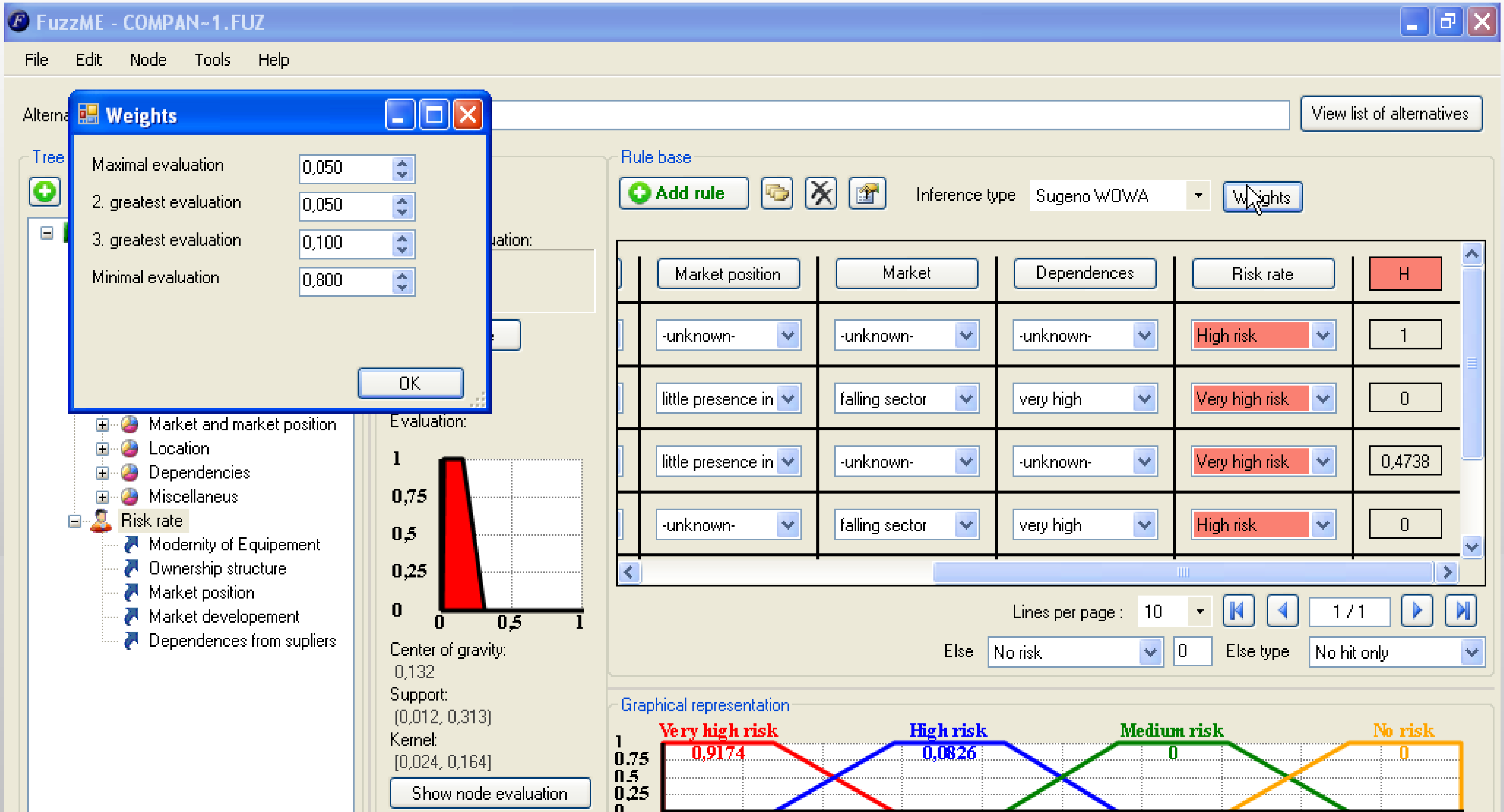
Application of FuzzME in banking

Fyzzy expert system aggregation, Sugeno-WOWA: *Risk Rate - A*



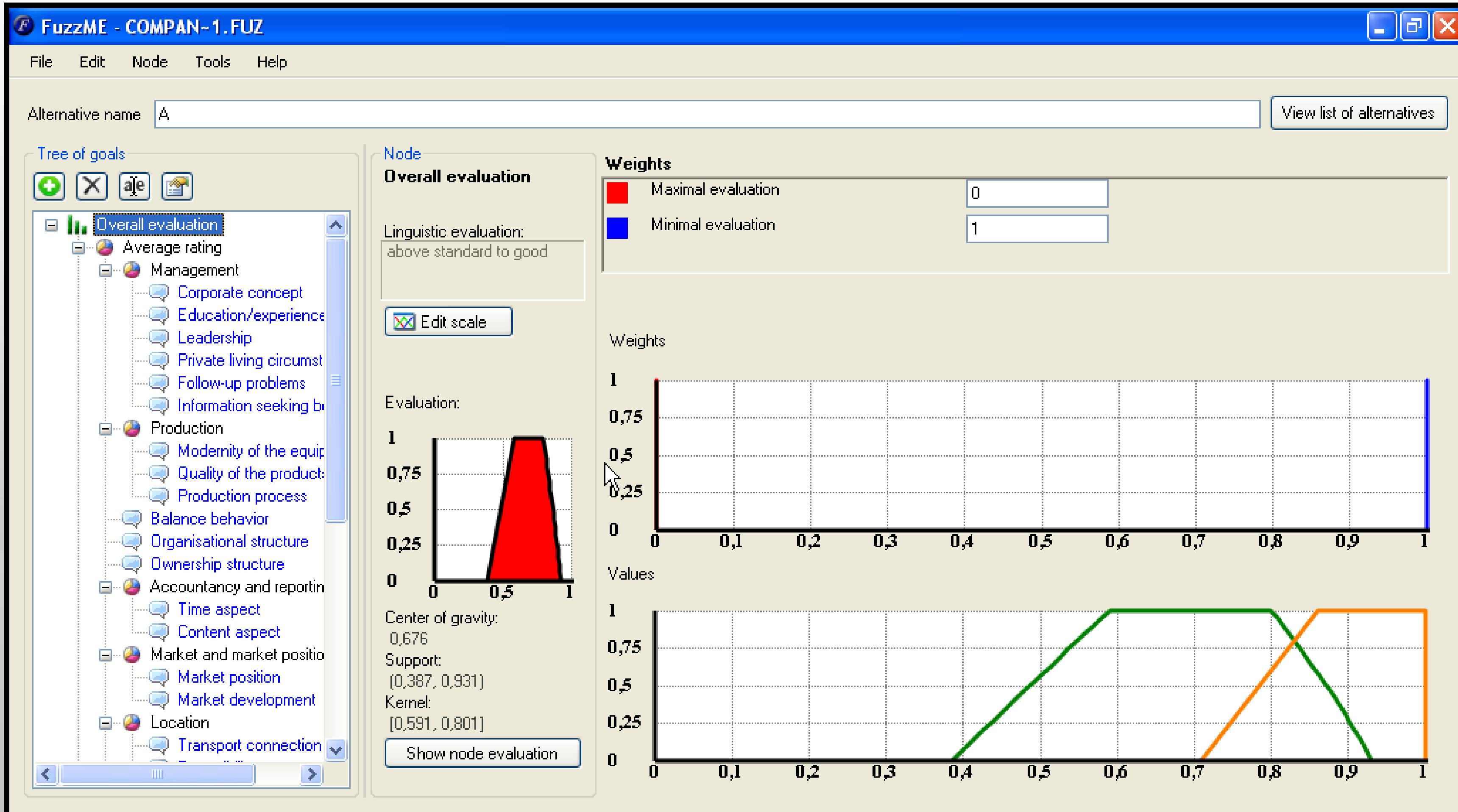
Application of FuzzME in banking

Fyzzy expert system aggregation, Sugeno WOWA: *Risk Rate - B*



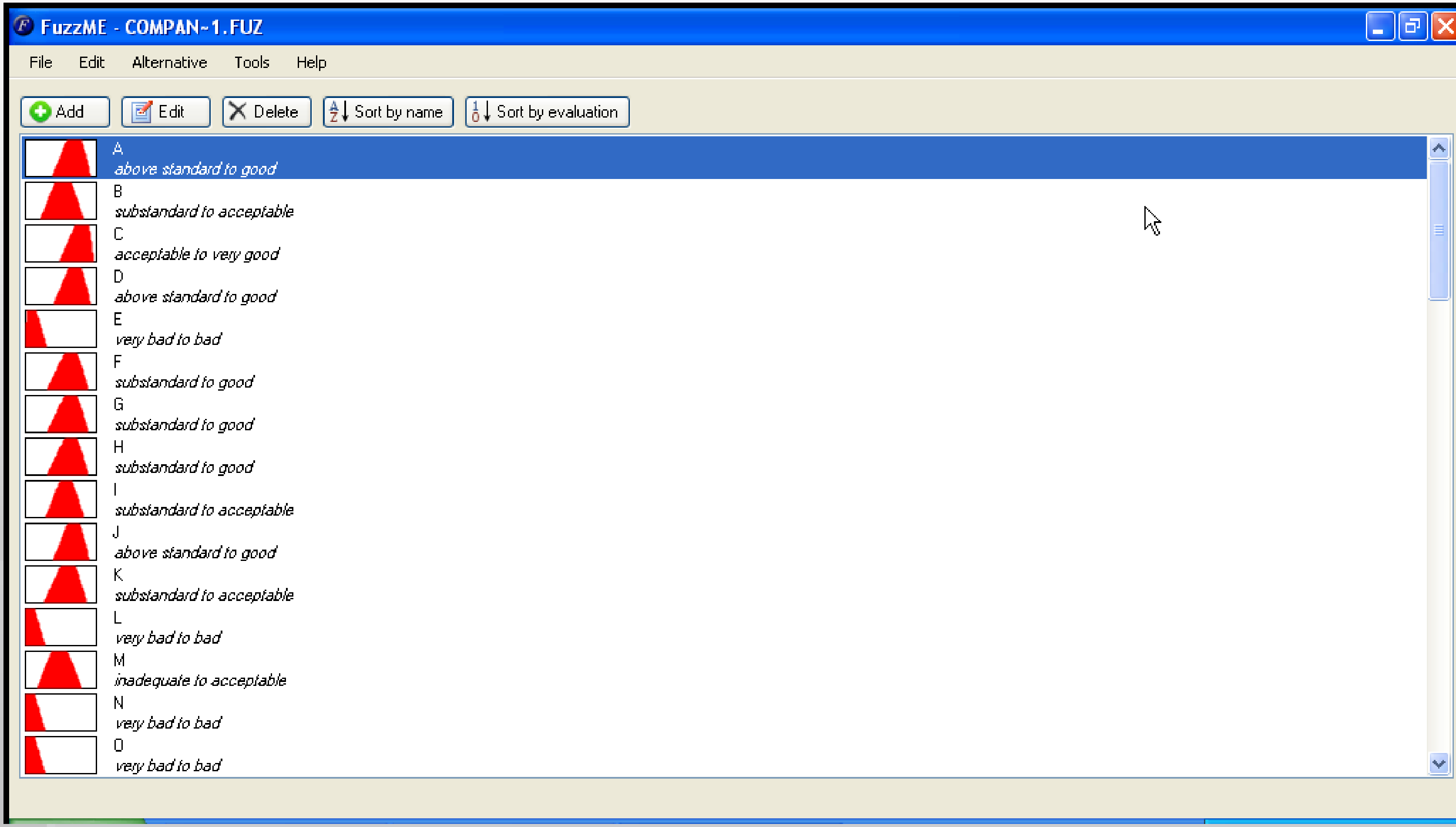
Application of FuzzME in banking

Ordered fuzzy weighted average aggregation – *Overall Evaluation*



Application of FuzzME in banking

Overall evaluations – linguistic approximation of results



Contacts

- A demo-version of the FuzzME software package:

  http://FuzzME.wz.cz/

References

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