# Two Approaches to Image Fusion

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# Outline

# Fusion of Images

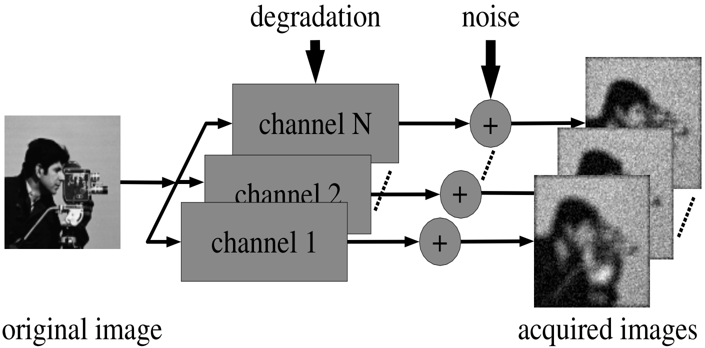
## Description of a Problem

- Ideal image $u$ – intensity function of two variables,
- $C_1, \dots, C_K$ – acquired channels,
- $C_i(x, y) = D_i(u(x, y)) + n_i(x, y)$ – image acquisition model where
    - $D_i$ – unknown operator describing the image degradations,
    - $n_i$ – additive random noise.

## Main Purpose of Image Fusion

To obtain an image $\hat{u}$ which is as a “good estimate” of $u$ and represents an original scene better than each individual channel.

# Multichannel acquisition model



# Piecewise Ideal Imaging (Multifocus Imaging)

- The degradation of each channel is given by a **convolution with space-invariant kernel**,
- every point $(x, y)$ of the scene is assumed to be acquired **undistorted in (at least) one channel**, i.e.

$$C_i(x, y) = (u * h_i^k)(x, y) \Leftrightarrow (x, y) \in \Omega_k,$$

where

$$h_i^k(x, y, s, t) = h_i^k(x - s, y - t),$$

$$\Omega = \bigcup_{i=1}^{K} \Omega_k,$$

$$(\forall k)(\exists i)\,(h_i^k(x - s, y - t) = \delta(x - s, y - t)).$$

# Image Fusion for Multifocus Imaging

## Idea of a Fusion Algorithm

- **Compare** the channels in image domain or in transformed domain.
- **Identify** the channel in which the pixel (or the region) is depicted undistorted, i.e.,
  - local focus measure is calculated over the pixel neighborhood,
  - the channel which maximizes the focus measure is chosen.
- **Mosaic** the undistorted parts.

# Most Popular Focus Measures

Focus measures are based on the quantity of **high frequencies**

- Image variance

$$M = \iint (C_i(x,y) - E_i)^2 \, \mathrm{dxdy}$$

  where $E_i$ denotes the mean gray level value of $C_i$.

- Energy of a Fourier spectrum

$$M = \iint |\hat{C}_i(u,v)| \, \mathrm{dudv}$$

# Image Fusion on the Base of the F-transform

**(1)** **Decompose** channel images $C_1, \dots, C_K$ into inverse F-transforms and error functions using the one-level decomposition.

**(2)** Apply the **fusion operator**

$$\kappa(x_1, \dots, x_K) = x_p, \text{ if } |x_p| = \max(|x_1|, \dots, |x_K|)$$

to the respective F-transform components of $C_i$, $i \in I$.

**(3)** Apply the **fusion operator** to the to the respective F-transform components of the error functions $e_i$, $i \in I$.

**(4)** **Reconstruct** the fused image from the inverse F-transforms with the fused components of the image and the fused components of the error function.

# Local Focus Measures for the F-transform Fusion

F-transform focus measures reflect quantity of **low frequencies**

$$M_1^{kl} = \iint (C_i(x,y) A_k(x) B_l(y) \,\mathrm{dxdy}$$

$$M_2^{pq} = \iint \left( \sum_{k=1}^{n} \sum_{l=1}^{m} (C_i(x,y) - M_1^{kl}) A_k(x) B_l(y) \right) A_p(x) B_q(y) \mathrm{dxdy}$$

# Properties of Local Focus Measures for the F-transform Fusion

- **Monotonicity**
- **Robustness**

## Robustness

A focus measure $M$ is **robust** if for any four images $I_1$, $I_2$, $\tilde{I}_1$ and $\tilde{I}_2$ such that $\tilde{I}_1$ and $\tilde{I}_2$ are "close" to $I_1$ and $I_2$, respectively,

$$M(I_1) < M(I_2) \Rightarrow M(\tilde{I}_1) < M(\tilde{I}_2).$$

# Robustness and Removing Noise

## Removing Additive Noise

- F-Transform **removes an additive noise** $s \in C[a, b]$ if

$$\mathbf{F}_{n,s} = [\mathbf{0}, \ldots, \mathbf{0}].$$

- In this case, for all $x \in [a, b]$

$$f_{F,n}(x) = (f + s)_{F,n}(x).$$

A focus measure $M_1$ for the F-transform Fusion is **robust** is the closeness is connected with a presence of an additive noise.

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# Removing Noise

## Which Noise can be Removed?

Noise $s$ is removable on $[x_2, x_{n-1}]$ if

- $s \in C[a,b]$ – $2h$-periodical function and for $k = 2, \ldots, n-1$

$$s(x_k - x) = -s(x_k + x) \qquad \text{on interval } [x_{k-1}, x_{k+1}],$$

or

- $s \in C[a,b]$ – $h$-periodical function and for $k = 2, \ldots, n-1$

$$\int_{x_{k-1}}^{x_k} s(x)\mathrm{dx} = 0.$$

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# Image Reconstruction

## Problem Description

**Image reconstruction** – reconstruction of a damaged image where the damage is anything what the original image does not include. It can be noise, text, scratch, etc.

# F-transform for Image Reconstruction

**Assumption.** The damaged area can be separated

## Proposed Method

- Apply the F-transform (approximation + filtration)
- Fuse the original (damaged) image with the inverse F-transform

# Illustration

# Conclusion

- Fusion is considered from the point of maximizing a local focus measure
- Traditional and the F-transform approaches has been discussed
- Properties of the F-transform focus measure were highlighted
- The F-transform based fusion for reconstruction was introduced

# Future Research

- Reduce one optimization step in the F-transform fusion
- Reduce manual choice of parameters in Fusion for Reconstruction