Internal operators: Application to image fusion

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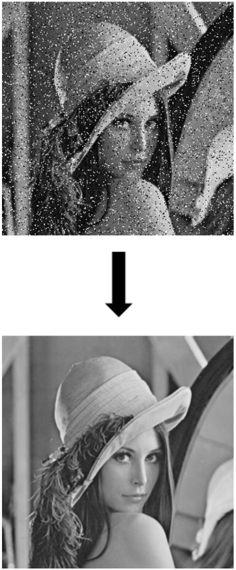
Outline

Impulsive noise

Impulsive noise

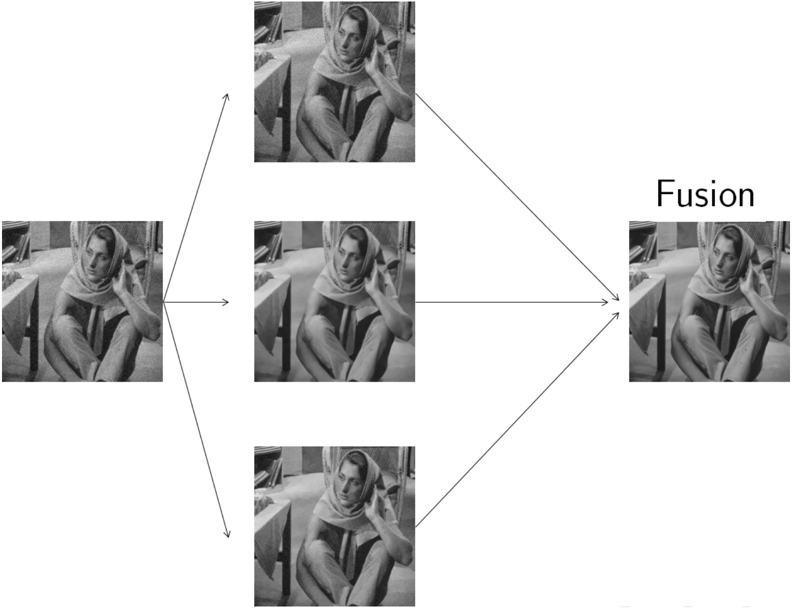


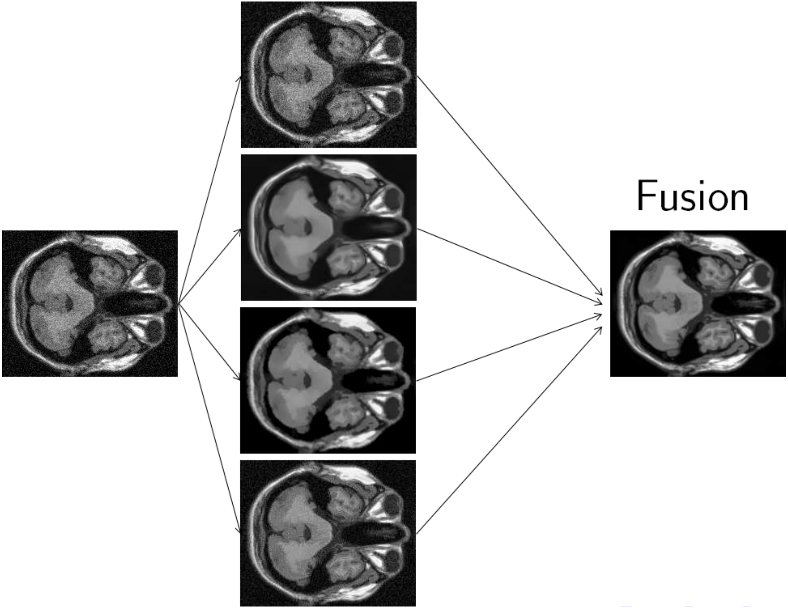
Impulsive noise

Additive noise


Impulsive noise
Additive noise

Any other noise...
Which filter?


Fusion


Fusion

Definition

An internal fusion operator is a mapping

$$IF : [0,1]^n \to [0,1]$$

such that $IF(x_1, \ldots, x_n) = x_j$ with $j \in \{1, \ldots, n\}$ for every $x_1, \ldots, x_n \in [0,1]$.

Proposition

Let $IF : [0,1]^n \to [0,1]$ be an internal fusion operator. Then, the following items hold:

1. *IF is idempotent:* $IF(x, \ldots, x) = x$ *for every* $x \in [0,1]$*;*
2. *IF is averaging:* $min(x_1, \ldots, x_n) \leq IF(x_1, \ldots, x_n) \leq max(x_1, \ldots, x_n)$ *for every* $x_1, \ldots, x_n \in [0,1]$.

Example

1. Let π_j denote the j-th projection; that is,

$$\pi_j(x_1, \ldots, x_n) = x_j$$

   Then for every $j = 1, \ldots, n$, π_j is n internal fusion operator.
2. Assume that p is an odd number ($n = 2k + 1$ for some $k \geq 1$). Then, the median operator is an internal fusion operator.
3. Both the maximum and the minimum are internal fusion operators.

However, other properties are not satisfied. An internal fusion operator needs not to be:

1. Homogeneous;
2. Shift-invariant;
3. Monotone;
4. Migrative;
5. ...

Let's denote by $\mathcal{IF}(n)$ the class of all internal fusion operators over $[0,1]^n$. Then we have the following result:

Proposition

$(\mathcal{IF}(n), \max, \min)$ *is a bounded lattice, with the operations* max *and* min *defined, for every* $F, G \in \mathcal{IF}$ *and* $(x_1, \ldots, x_n) \in [0,1]^n$*, as:*

$$\max(F,G)(x_1,\ldots,x_n) = \max(F(x_1,\ldots,x_n), G(x_1,\ldots,x_n))$$

and

$$\min(F,G)(x_1,\ldots,x_n) = \min(F(x_1,\ldots,x_n), G(x_1,\ldots,x_n))$$

Proposition

Let's denote by

$$F_\infty = \sup\{F : [0,1]^n \to [0,1] | F \in \mathcal{IF}(n)\}$$

and

$$F_0 = \inf\{F : [0,1]^n \to [0,1] | F \in \mathcal{IF}(n)\}$$

Then $F_\infty(x_1, \ldots, x_n) = \max(x_1, \ldots, x_n)$ *and* $F_0(x_1, \ldots, x_n) = \min(x_1, \ldots, x_n)$.

Proposition

Let $F, G_1, \ldots, G_n \in \mathcal{IF}(n)$ *be internal fusion operators. Let's denote by* $F \circ G$ *the operator*

$$F \circ G(x_1, \ldots, x_n) = F(G_1(x_1, \ldots, x_n), \ldots, G_n(x_1, \ldots, x_n))$$

Then $F \circ G$ *is also an internal fusion operator.*

Definition

Consider a family of indexes I. A family $\{\varphi_i\}_{i\in I}$ with $\varphi : [0,1]^n \to \{0,1\}$ for every $i \in I$ is a partition if for every $(x_1, \ldots, x_n) \in [0,1]^n$ there exists $i_0 \in I$ such that $\varphi_{i_0}(x_1, \ldots, x_n) = 1$ and $\varphi_i(x_1, \ldots, x_n) = 0$ for every $i \neq i_0$.

Theorem

A mapping $F : [0,1]^n \to [0,1]$ *is an internal fusion operator of dimension n if and only if there exists a partition of* $[0,1]^n$ $\{\varphi_1, \ldots, \varphi_n\}$ *such that* $F(x_1, \ldots, x_n) = \varphi_1 \pi_1(x_1, \ldots, x_n) + \cdots + \varphi_n \pi_n(x_1, \ldots, x_n)$.

Corollary

Let $F : [0,1]^n \to [0,1]$ *be an internal fusion operator. Then, if the identities*

$$F(x_1,\ldots,x_n) = \varphi_1\pi_1(x_1,\ldots,x_n) + \cdots + \varphi_n\pi_n(x_1,\ldots,x_n)$$

and

$$F(x_1,\ldots,x_n) = \psi_1\pi_1(x_1,\ldots,x_n) + \cdots + \psi_n\pi_n(x_1,\ldots,x_n)$$

hold for every $(x_1,\ldots,x_n) \in [0,1]^n$ *such that* $(x_i \neq x_j$ *whenever* $i \neq j$, *where* $\{\varphi_j\}_{j=1,\ldots,n}$ *and* $\{\psi_j\}_{j=1,\ldots,n}$ *are partitions of* $[0,1]^n$, *then it follows that* $\varphi_j = \psi_j$ *for every* $j = 1,\ldots,n$.