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Advances in F-transform based image fusion

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INVESTICE DO VAŠÍ BUDOUCNOSTI

OP Výzkum a vývoj pro inovace

Outline

Seminal paper on F-transform:

[1] Perfilieva, I., Fuzzy Transform: Theory and Applications. Fuzzy sets and systems, 2006, Vol. 157 (8), 992–1023.

Based on results obtained with Radek Valášek:

[2] Daňková, M., Valášek, R.. Full Fuzzy Transform and the Problem of Image Fusion. Journal of electrical engineering. 2006, Vol. 12, 82–84.

and subsequently followed by

[3] Perfilieva, I., Daňková, M. Image Fusion on the Basis of Fuzzy Transforms. Computational Intelligence in Decision and Control. New Jersey: World Scientific, 2008, 471-476.

Image fusion – problem specification

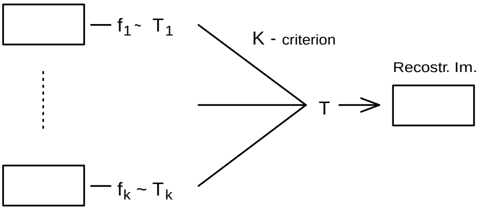
- What?

  "k" partially damaged images → one "good" image

- Tool?

  Aggregation operators, F-transform, Mathematical morphology, Wavelet transform

- How (Principle)?



F-transform tool

Let u be represented by the function $u : P \to \mathbb{R}$ of two variables where $P = \{(i, j) \mid i = 1, \ldots, N, j = 1, \ldots, M\}$ is an $N \times M$ array of pixels and $\mathbb{R}$ is the set of reals.

- Partition
  $A_1, \ldots, A_n \underset{\sim}{\subset} [1, N]$ establish a *fuzzy partition* of $[1, N]$ if the following requirements are fulfilled:
  (i) Locality: for every $k = 1, \ldots, n$, $A_k(x) = 0$ if $x \in [1, N] \setminus [x_{k-1}, x_{k+1}]$ where $x_0 = x_1$, $x_{N+1} = x_N$;
  (ii) Continuity: for every $k = 1, \ldots, n$, A_k is continuous on $[x_{k-1}, x_{k+1}]$ where $x_0 = x_1$, $x_{N+1} = x_N$;
  (iii) Ruspini: for every $i = 1, \ldots, N$, $\sum_{k=1}^{n} A_k(i) = 1$;
  (iv) Non-emptiness: for every $k = 1, \ldots, n$, $\sum_{i=1}^{N} A_k(i) > 0$.

F-transform tool – Direct F-transform

Let $u : P \to \mathbb{R}$ and fuzzy sets $A_k \subsetneqq [1, N], B_l \subsetneqq [1, M]$, $k = 1, \ldots, n$, $l = 1, \ldots, m$ establish a fuzzy partition of $[1, N] \times [1, M]$.

- Direct F-transform of u is an image of the mapping $F[u] : \{A_1, \ldots, A_n\} \times \{B_1, \ldots, B_m\} \to \mathbb{R}$ defined by

$$F[u](A_k, B_l) = \frac{\sum_{i=1}^{N} \sum_{j=1}^{M} u(i,j) A_k(i) B_l(j)}{\sum_{i=1}^{N} \sum_{j=1}^{M} A_k(i) B_l(j)}, \tag{1}$$

- Each component $F[u]_{kl}$ is a local mean value of u over a support set of the respective fuzzy sets (A_k, B_l).

F-transform tool – Inverse F-transform

Components $F[u]_{kl}$ can be arranged into the matrix or vector representation as follows:

$$(F[u]_{11}, \ldots, F[u]_{1m}, \ldots, F[u]_{n1}, \ldots, F[u]_{nm}).$$

- Inverse F-transform of u is a function on P which is represented by the following inversion formula where $i = 1, \ldots, N, j = 1, \ldots, M$:

$$u_{nm}(i,j) = \sum_{k=1}^{n} \sum_{l=1}^{m} F[u]_{kl} A_k(i) B_l(j). \tag{2}$$

Properties of F-transform

Let $[a, b]$ be h-uniformly partitioned by $A_1, \dots, A_n$, where $n > 2$ and $h = (b - a)/(n - 1)$,
f be a continuous function on $[a, b]$,
$F_1, \dots, F_n$ be the F-transform components of f w.r.t. $A_1, \dots, A_n$.

P1. The k-th component F_k $(k = 1, \dots, n)$ minimizes the function

$$\Phi(y) = \int_a^b (f(x) - y)^2 A_k(x) dx.$$

Properties of F-transform

P2. For each $k = 1, \ldots, n-1$, and for each $t \in [x_k, x_{k+1}]$ the following estimations hold:

$$|f(t) - F_k| \leq 2\omega(h, f), \qquad |f(t) - F_{k+1}| \leq 2\omega(h, f)$$

where

$$\omega(h, f) = \max_{|\delta| \leq h} \max_{x \in [a, b-\delta]} |f(x+\delta) - f(x)|$$

is the modulus of continuity of f on $[a, b]$.

Properties of F-transform

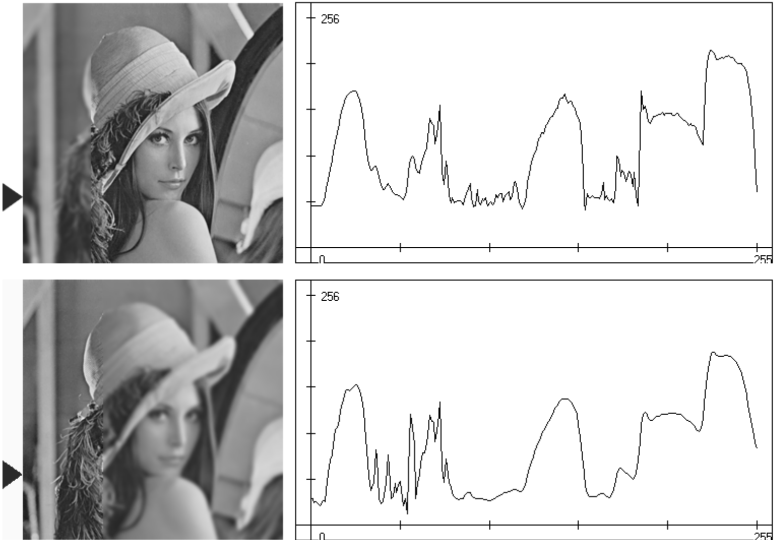
P3. F-transform of a continuous periodical function f on $[a, b]$ with period $2h$ such that

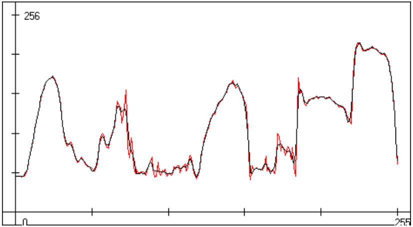
$$f(x_k - x) = -f(x_k + x), \quad x \in [x_{k-1}, x_{k+1}].$$

Then all components of the direct F-transform are equal to zero.

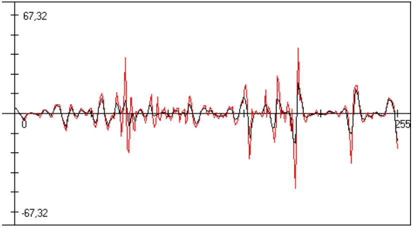
- F-transform can remove a noise with the above characteristic.

Motivation for the use of F-transform

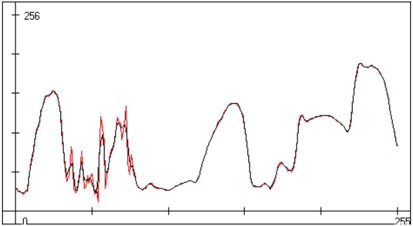




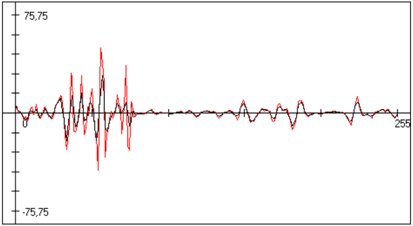
(a) $f^{(1)}$ and $f^{(1)}_{128}$



(b) $f^{(1)} - f^{(1)}_{128}$ and its approximation



(c) $f^{(2)}$ and $f^{(2)}_{128}$



(d) $f^{(2)} - f^{(2)}_{128}$ and its approximation

Expected result of fusion using F-transform

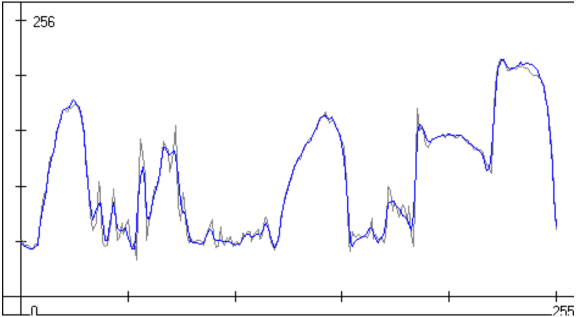


Figure: Fused function.

Justification of using F-transform

- By **P2**, smaller the modulus of continuity higher the quality of approximation of an input image by its inverse fuzzy transform.
- If a certain part of an input image is affected by degradation producing small variations between functional values in a close neighborhood, then by **P1**, the respective fuzzy transform component capture the weighted arithmetic mean and the error function is close to zero at that part.

Decomposition for image fusion

- By decomposition we extract relevant information.
- We distinguish One-level and Higher-level image decompositions.

Further, we deal with a real-valued function $u = u(x, y)$ defined on the $N \times M$ array of pixels
$P = \{(i, j) \mid i = 1, \ldots, N, j = 1, \ldots, M\}$ so that $u : P \to \mathbb{R}$.

One-level decomposition

We stem from the following representation of u on P:

$$u(x,y) = u_{nm}(x,y) + e(x,y), \text{ where } 0 < n \leq N, 0 < m \leq M,$$
$$e(x,y) = u(x,y) - u_{nm}(x,y), \forall (x,y) \in P.$$

where $n < N$, $m < M$, u_{nm} is the inverse F-transform of u and e is the respective residuum.

Image = F-transform of Image + Error

Higher-level decomposition

We decompose the error function e using its inverse F-transform with respect to a finer fuzzy partition of P that consists of $n' : n < n' \leq N$ and $m' : m < m' \leq M$ basic functions respectively. Thus we will obtain 2-level decomposition

$$u(x,y) = u_{nm}(x,y) + e_{n'm'}(x,y) + e'(x,y),$$
$$e'(x,y) = e(x,y) - e_{n'm'}(x,y), \ \forall (x,y) \in P.$$

Image = F-transform of Image + F-transform of Error + Error

Higher-level decomposition

The higher-level decomposition ($(k-1)$-level) formula:

$$u(x,y) = u_{n_1 m_1}(x,y) + \sum_{r=1}^{k-2} e^{(r)}_{n_{r+1} m_{r+1}}(x,y) + e^{(k-1)}(x,y),$$

where

$$\begin{aligned}
&0 < n_1 \leq n_2 \leq \ldots \leq n_{k-1} \leq N,\\
&0 < m_1 \leq m_2 \leq \ldots \leq m_{k-1} \leq M,\\
&e^{(1)}(x,y) = u(x,y) - u_{n_1 m_1}(x,y),\\
&e^{(i)}(x,y) = e^{(i-1)}(x,y) - e^{(i-1)}_{n_i m_i}(x,y),\\
&\text{for } i = 2,\ldots,k-1 \text{ and } (x,y) \in P.
\end{aligned}$$

F-transform based image fusion algorithms

There are two algorithms:

1. Simple F-transform based image fusion algorithm – based on one-level decomposition.
2. Complete F-transform based algorithm – based on higher-level decomposition.

Fusion operator $\kappa : \mathbb{R}^K \to \mathbb{R}$ which is defined as follows:

$$\kappa(x_1, \dots, x_K) = x_p, \text{ if } |x_p| = \max(|x_1|, \dots, |x_K|). \tag{3}$$

Using κ we chose components with maximal absolute values, which should correspond to parts of the least degraded input images.

Simple algorithm

Based on one-level decomposition

(1) Decompose input images into inverse F-transforms and error functions using one-level decomposition formula.

(2) Apply fusion operator to the inverse F-transforms to produce a fused F-transform.

(3) Apply fusion operator to the error functions to produce a fused error function.

(4) Reconstruct the fused image from the fused F-transform and the fused error function.

Complete algorithm

Based on higher-level decomposition

(1) Decompose input images into inverse F-transforms and error functions using higher-level decomposition formula.
(2) Apply fusion operator to the inverse F-transforms to produce a fused F-transforms.
(3) Apply fusion operator to the error functions to produce a fused error function.
(4) Reconstruct the fused image from the fused F-transforms and the fused error function.

Complexity

- A computational complexity of CA is higher than SA.
- Complexity of an inverse F-transform is

$$\mathcal{O}(N \cdot M)$$

- CA uses higher number of F-transforms multiplied by the number of images K.
- And finally, there is the fusion operator, which compares $Kk_{\max}mn + KMN$ values.

Hence, a speed of CA depends mainly on the number of iterations processed in the algorithm.

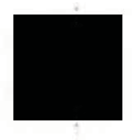
Fusion of blurred images

(a) 1'st blurred image (b) 2'nd blurred image (c) O – Original image

Fusion of blurred images

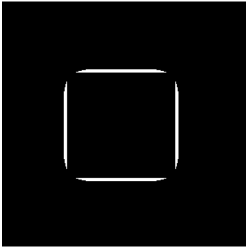
(d) F_1 – fused image by SA with $n, m = 1$

(e) F_2 – fused image by SA with $n, m = 9$

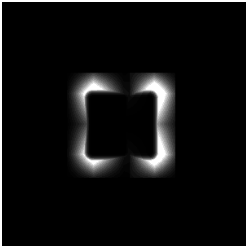


(f) F_3 – fused image by CA with $n_{start}, m_{start} = 1, k_{\max} = 8, step = 2$

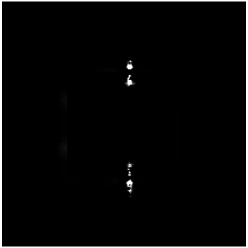
Fusion of blurred images



(g) $|O - F_1|$



(h) $|O - F_2|$



(i) $|O - F_3|$

Fusion of multi-channel color images

(j) C_1 – 1'st image (background in focus)

(k) C_2 – 2'nd image (toy in focus)

Fusion of multi-channel color images

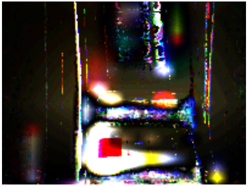
(l) F_1 – fused image by CA with $n_{start}, m_{start} = 1, k_{\max} = 5, step = 2$

(m) F_2 – fused image by CA with $n_{start}, m_{start} = 10, k_{\max} = 4, step = 2$

Fusion of multi-channel color images

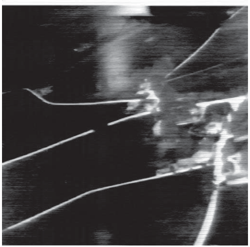
(n) R – region of interest cut from C_1

(o) $|R - R_{F_1}|$



(p) $|R - R_{F_2}|$

Fusion of multi-sensor images

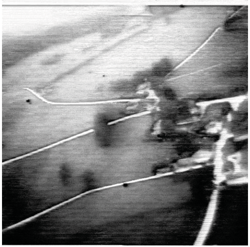


(q) FLIR image

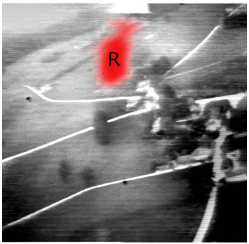
(r) LLTV image

(s) Fusion by means of multiresolution analysis

Fusion of multi-sensor images



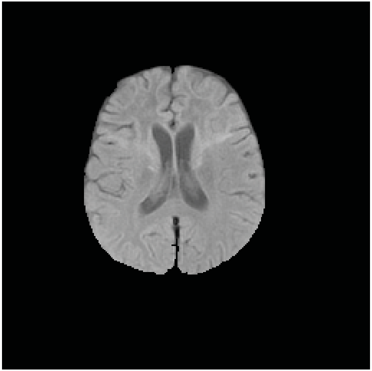
(t) SA $n, m = 100$



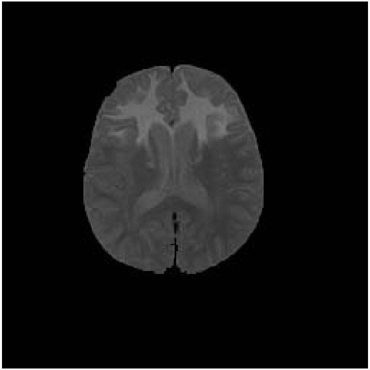
(u) Problematic part R

(v) CA with $n_{start}, m_{start} = 40$, $k_{max} = 3$, $step = 2$

Fusion of multi-sensor images



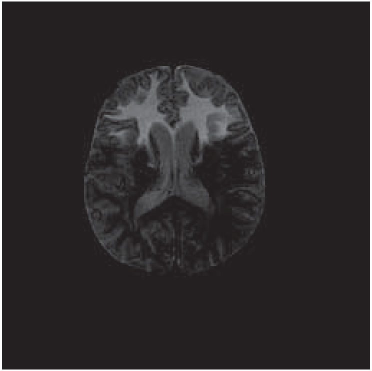
(w) 1'st MRI image



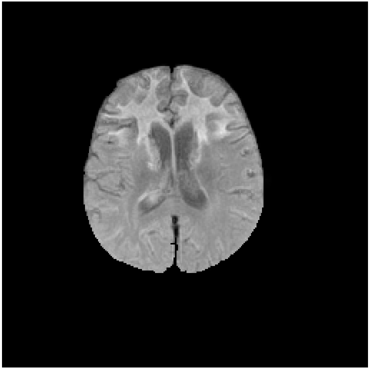
(x) 2'nd MRI image

Figure: One axial slice of dual-echo magnetic resonance imaging acquisitions (pathological brain image), Courtesy Professor Catherine Adamsbaum, Saint Vincent de Paul Hospital, Paris.

Fusion of multi-sensor images



(a) 2'nd image with modified contrast



(b) CA with $n_{start}, m_{start} = 10$, $k_{max} = 5$, $step = 2$

F-transform tool

Let $\mathcal{L} = \langle [0,1], *, \rightarrow, \wedge, \vee, 0, 1 \rangle$ be a residuated lattice.

- Partition

  $A_1, \ldots, A_n \underset{\sim}{\subset} [1, N]$ establish a *fuzzy partition* of $[1, N]$ if the following requirements are fulfilled:

  (i) Locality: for every $k = 1, \ldots, n$, $A_k(x) = 0$ if $x \in [1, N] \setminus [x_{k-1}, x_{k+2}]$ where $x_0 = x_1$, $x_{N+1} = x_N$;

  (ii) Continuity: for every $k = 1, \ldots, n$, A_k is continuous on $[x_{k-1}, x_{k+2}]$ where $x_0 = x_1$, $x_{N+1} = x_N$;

  (iii) Ruspini-like: for every $i = 1, \ldots, N$, $\bigvee_{k=1}^{n} A_k(i) = 1$;

  (iv) Non-emptiness: for every $k = 1, \ldots, n$, $\sum_{i=1}^{N} A_k(i) > 0$.

Let $u : P \rightarrow [0, 1]$ and fuzzy sets $A_k \underset{\sim}{\subset} [1, N]$, $B_l \underset{\sim}{\subset} [1, M]$, $k = 1, \ldots, n$, $l = 1, \ldots, m$ establish a fuzzy partition of $[1, N] \times [1, M]$.

Direct and inverse F-transform

- Direct F-transform of u is an image of the mapping $F[u] : \{A_1, \ldots, A_n\} \times \{B_1, \ldots, B_m\} \to \mathbb{R}$ defined by

$$F[u](A_k, B_l) = \bigwedge_{i=1}^{N} \bigwedge_{j=1}^{M} [A_k(i) * B_l(j)] \to u(i, j), \tag{4}$$

- Inverse F-transform of u is a function on P which is represented by the following inversion formula where $i = 1, \ldots, N, j = 1, \ldots, M$:

$$u_{nm}(i, j) = \bigvee_{k=1}^{n} \bigvee_{l=1}^{m} [A_k(i) * B_l(j) * F[u]_{kl}]. \tag{5}$$

Properties of F-transform

P1. Each component $F[u]_{kl}$ is a local minimum value of u over a support set of the respective fuzzy sets (A_k, B_l).

P2. $u_{nm}(i,j) \leq u(i,j), \ \forall (i,j)$ - lower approximation.

P3. $F[u]_{kl}, \ \forall k, l$ are the best components from all g_{kl} satisfying

$$\bigvee_{k=1}^{n} \bigvee_{l=1}^{m} [A_k(i) * B_l(j) * g_{kl}] \leq u(i,j).$$

One-level decomposition

We stem from the following representation of u on P:

$$u(x, y) = u_{nm}(x, y) + e(x, y), \text{ where } 0 < n \leq N, 0 < m \leq M,$$
$$e(x, y) = u(x, y) - u_{nm}(x, y), \forall(x, y) \in P.$$

where $n < N$, $m < M$, u_{nm} is the inverse F-transform of u and e is the respective residuum.

Image = F-transform of Image + Error

Simple algorithm

Based on one-level decomposition

(1) Decompose input images into inverse F-transforms and error functions using one-level decomposition formula.

(2) Apply fusion operator to the inverse F-transforms to produce a fused F-transform.

(3) Apply fusion operator to the error functions to produce a fused error function.

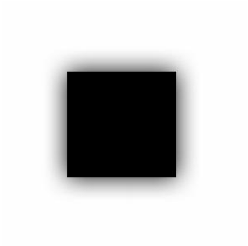
(4) Reconstruct the fused image from the fused F-transform and the fused error function.

Fusion operator $\kappa : \mathbb{R}^K \to [0, 1]$ which is defined as follows:

$$\kappa(x_1, \ldots, x_K) = \max(x_1, \ldots, x_K). \tag{6}$$

Using κ we chose components with maximal values.

Fusion by F-transform over a residuated lattice



(c) Fused squares

(d) Fused toy

(e) Fused road

Simple algorithm - generally

Based on one-level decomposition

(1) Decompose input images into inverse F-transforms and error functions using one-level decomposition formula.

(2) Apply 1'st fusion operator to the inverse F-transforms to produce a fused F-transform.

(3) Apply 2'nd fusion operator to the error functions to produce a fused error function.

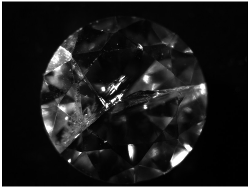
(4) Reconstruct the fused image from the fused F-transform and the fused error function.

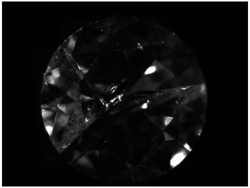
Fusion operators $\kappa_{1,2} : \mathbb{R}^K \to [0,1]$ – aggregation operator that can be defined e.g. as

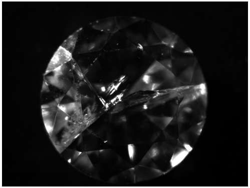
$$\kappa_1(x_1, \dots, x_K) = \min(x_1, \dots, x_K),$$
$$\kappa_2(x_1, \dots, x_K) = \max(x_1, \dots, x_K).$$

Using $\kappa_{1,2}$ we aggregate components that fits our purposes.

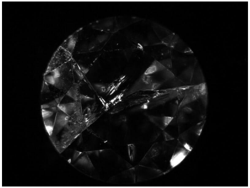
Fusion using various fusion operators


(f) Stone 1


(g) Stone 2


(h) Stone 3

Fusion using various fusion operators



(i) Fusion 1

(j) Fusion 2

(k) Fusion 3

Conclusions

- +++ SA proceeds very fast.
- - SA demands manual setting.
- - CA rapidly slow down with high number of iterations.
- + CA provides highly desirable result with maximal setting.
- + higher contrast elements are propagated.
- - by P3. SA and CA increase noise from input images.

Hence, the optimal choice is

SA with preset numbers of components $(N/2, M/2)$.