Distributive MV-algebra Pastings

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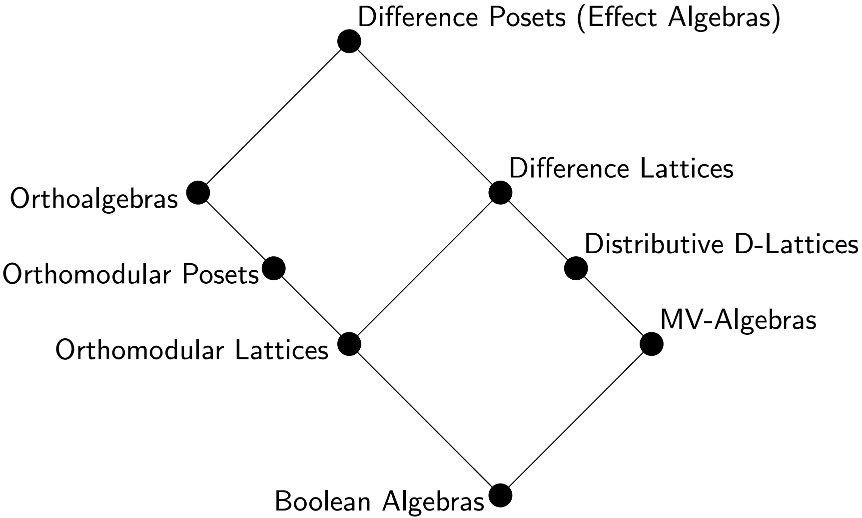
Algebraic structures



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In Greechie logics Boolean algebras generate blocks with the intersection of each pair of blocks containing at most one atom.

Greechie diagrams

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Vertices are drawn as points or small black circles and edges as smooth lines connecting atoms belonging to a block.

Greechie diagrams

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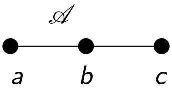
If a Boolean algebra $\mathscr{A}$ contains n atoms (the Greechie diagram of $\mathscr{A}$ consists of n vertices lying on one line), then $\mathscr{A}$ is isomorfic to the power set of a set with n elements, thus the cardinality of $\mathscr{A}$ is 2^n.

Greechie and Hasse diagrams of a Boolean algebra



Greechie diagram of the Boolean algebra $\mathscr{A}$

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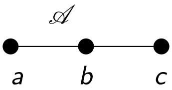

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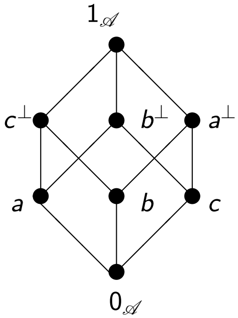

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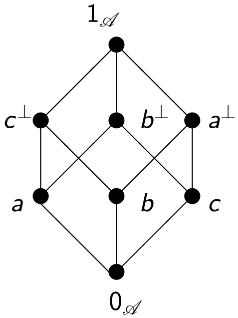

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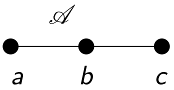

Greechie diagram of the Boolean algebra $\mathscr{A}$

$$At(\mathscr{A}) = \{a, b, c\}$$

$$|\mathscr{A}| = 2^3 = 8$$

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Loop Lemma (Greechie)

A Greechie logic $\mathscr{G}$ is

- an orthomodular poset iff $\mathscr{G}$ has no 3-loops,
- an orthomodular lattice iff $\mathscr{G}$ has no 3-loops and 4-loops.

Loops in Greechie logics

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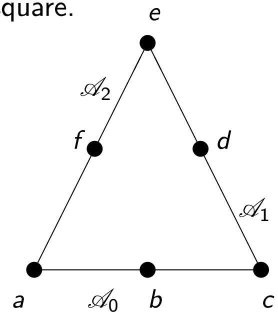

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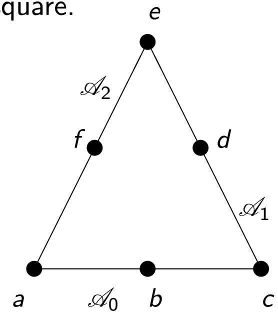
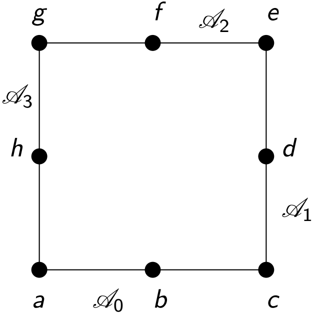


3-**loop**



4-**loop**

Pasting of Boolean algebras

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NAVARA, M.: *Constructions of quantum structures*, In: Handbook of Quantum Logic and Quantum Structures: Quantum Structures. (Eds. - K. Engesser, D. M. Gabbay, D. Lehmanm) Elsevier 2007, pp. 335-366

MV-algebra pasting

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These efforts were successful only after Riečanová proved that every lattice-ordered effect algebra (D-lattice) is a set-theoretical union of maximal sub-D-lattices of pairwise compatible elements, i.e. maximal sub-MV-algebras.

RIEČANOVÁ, Z.: *Generalization of blocks for D-lattices and lattice-ordered effect algebras*, Inter. J. Theor. Phys. **39** (2000), 231-237

MV-algebra pasting

A method of a construction of difference lattices by means of an MV-algebra pasting was originally suggested in

CHOVANEC, F., JUREČKOVÁ, M.: *MV-algebra pastings*, Inter. J. Theor. Phys. **42** (2003), 1913-1926,

but it was later revealed that some notions were not formulated correctly and they have been reformulated in the following manuscript

CHOVANEC, F.: *Graphic Representation of MV-algebra Pastings* , submitted to Mathematica Slovaca.

MV-algebra pasting

In this contribution we do not dealing with the conditions of the pasting of MV-algebras, but we assume that this pasting is already given.

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We write

$$\mathscr{M} = \mathscr{M}(\tau(a_1), \tau(a_2), \ldots, \tau(a_n))$$

and

$$|\mathscr{M}| = (\tau(a_1)+1)(\tau(a_2)+1)\ldots(\tau(a_n)+1).$$

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In this case we denote a vertex **a** of a Greechie diagram in the form $\mathbf{a}(\tau(\mathbf{a}))$, where **a** is an atom and $\tau(\mathbf{a})$ is its isotropic index.

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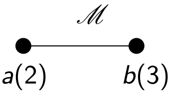
Then a finite MV-algebra is uniquely determined by its Greechie diagram.

Greechie and Hasse diagrams of an MV-algebra $\mathscr{M} = \mathscr{M}(2,3)$



Greechie diagram of the MV-algebra $\mathscr{M}$

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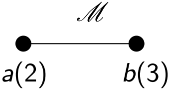

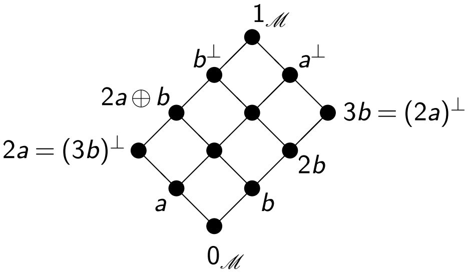

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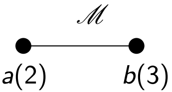

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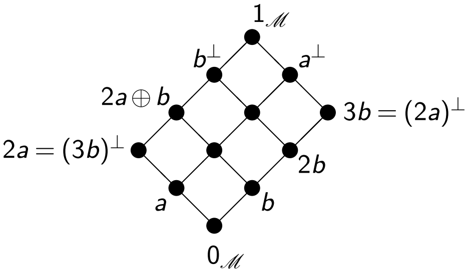

Hasse diagram of the MV- algebra $\mathscr{M}$

$$At(\mathscr{M}) = \{a, b\}, \quad \tau(a) = 2,\ \tau(b) = 3$$

$$|\mathscr{M}| = 3.4 = 12$$

Cluster Greechie diagrams

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Definition

Let $\mathscr{P}$ be an MV-algebra pasting. A **cluster Greechie diagram** (a CG-diagram for short) is a hypergraph $(\mathscr{V}, \mathscr{E})$, where $\mathscr{V}$ (the set of vertices) is a system of pairwise disjoint subsets of $At(\mathscr{P})$ such that $\bigcup \mathscr{V} = At(\mathscr{P})$ and $\mathscr{E}$ (the set of edges) is a system of sets of atoms of individual blocks in $\mathscr{P}$.

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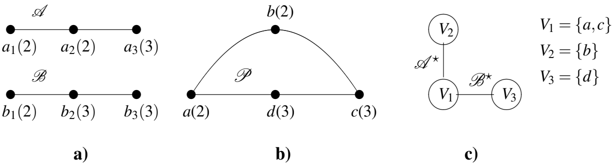
Vertices of a CG-diagram are drawn as small circles and edges as smooth lines connecting all sets of atoms belonging to a block.



a) - Greechie diagrams of MV-algebras $\mathscr{A}$ and $\mathscr{B}$

b) - Greechie diagram of the MV-algebra pasting $\mathscr{P} = \mathscr{A}^\star \cup \mathscr{B}^\star$, where

$(A,B) \in \mathscr{U}$, $A = \{a_1, a_3\}, B = \{b_1, b_3\}$

c) - CG- diagram of the MV-algebra pasting $\mathscr{P}$

3-loops in MV-algebra pastings

We define loops in MV-algebra pastings in a similar way as in Greechie logics.

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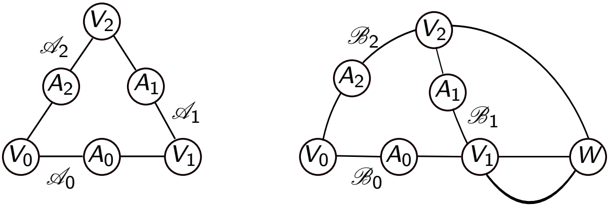

$At(\mathscr{A}_0) = V_0 \cup A_0 \cup V_1,\ \ At(\mathscr{A}_1) = V_1 \cup A_1 \cup V_2,$
$At(\mathscr{A}_2) = V_0 \cup A_2 \cup V_2$

$At(\mathscr{B}_0) = V_0 \cup A_0 \cup V_1 \cup W,\ At(\mathscr{B}_1) = V_1 \cup A_1 \cup V_2 \cup W,$
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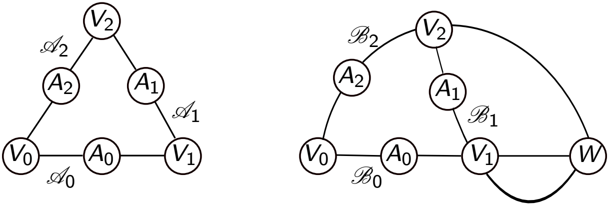
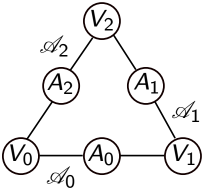
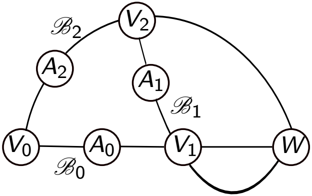


Atoms belonging to the sets V_i, $i = 0, 1, 2$, are called **nodal vertices**, and atoms belonging to the set W are called **central nodal vertices** of the 3-loop.





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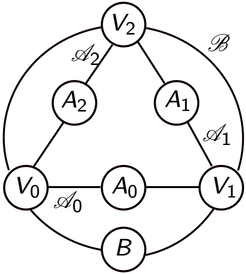
A 3-loop is difference poset that is not lattice-ordered.

3-loops in MV-algebra pastings

The following CG-diagram shows a pasting of four MV-algebras, where the blocks $\mathscr{A}_0$, $\mathscr{A}_1$, $\mathscr{A}_2$ create a 3-loop and the block $\mathscr{B}$ contains all nodal vertices of this 3-loop.



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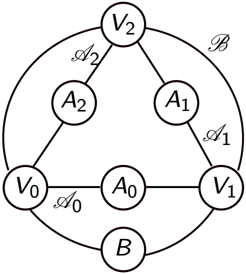

This pasting is a difference lattice (a D-lattice).

3-loops in MV-algebra pastings

Definition

We say that a 3-loop is **unbound** in an MV-algebra pasting $\mathscr{P}$, if there is no block in $\mathscr{P}$ containing all its nodal vertices.

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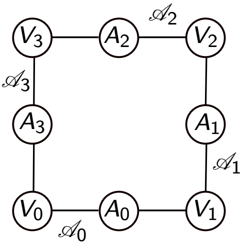
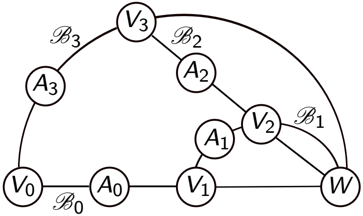
Theorem

Every MV-algebra pasting containing an unbound 3-loop is a D-poset that is not lattice-ordered.

4-loops in MV-algebra pastings

A pasting of four MV-algebras is called a **4-loop** (or a **loop of order 4**) if its cluster Greechie diagram is square shaped.





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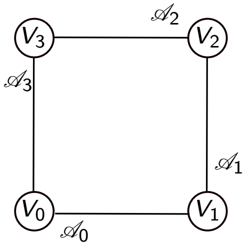
V_0, V_1, V_2, V_3 – sets of nodal vertices

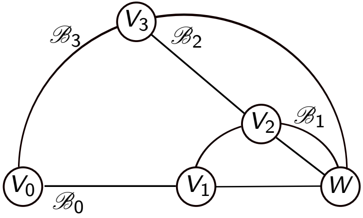
W – a set of central nodal vertices

Astroids

A 4-loop generated only by nodal vertices is called an **astroid**.

$At(\mathscr{A}_i) = V_i \cup V_{i+1} \cup W$ for every $i = 0, 1, 2, 3 \pmod 4$





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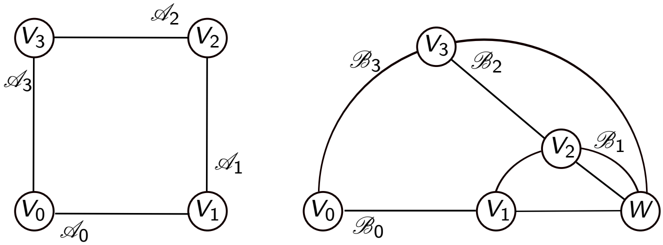

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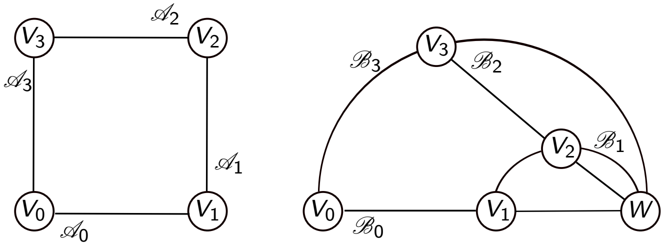

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Theorem

Let $\mathscr{P}$ be a 4-loop that is not an astroid. Then $\mathscr{P}$ is a D-poset that is not lattice-ordered.

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Let $\mathscr{P}$ be a pasting of MV-algebras containing a 4-loop $\mathscr{A}_0, \mathscr{A}_1, \mathscr{A}_2, \mathscr{A}_3$ and no unbound 3-loop. Let V_i $(i = 0, 1, 2, 3)$ be the sets of nodal vertices and W be the set of central nodal vertices of the 4-loop. Then $\mathscr{P}$ is a D-lattice if one of the following conditions is fulfilled:

Now we give some sufficient conditions for MV-algebra pasting containing a 4-loop, to be a D-lattice.

Let $\mathscr{P}$ be a pasting of MV-algebras containing a 4-loop $\mathscr{A}_0, \mathscr{A}_1, \mathscr{A}_2, \mathscr{A}_3$ and no unbound 3-loop. Let V_i $(i = 0, 1, 2, 3)$ be the sets of nodal vertices and W be the set of central nodal vertices of the 4-loop. Then $\mathscr{P}$ is a D-lattice if one of the following conditions is fulfilled:

(1) The 4-loop $\mathscr{A}_0, \mathscr{A}_1, \mathscr{A}_2, \mathscr{A}_3$ is an astroid.

(2) There is an astroid $\mathscr{B}_0, \mathscr{B}_1, \mathscr{B}_2, \mathscr{B}_3$ in $\mathscr{P}$ such that $V_i \subset U_i$ and $W \subset W_0$, where U_i $(i = 0, 1, 2, 3)$ are the sets of nodal vertices and W_0 is the set of central nodal vertices of the astroid $\mathscr{B}_0, \mathscr{B}_1, \mathscr{B}_2, \mathscr{B}_3$.

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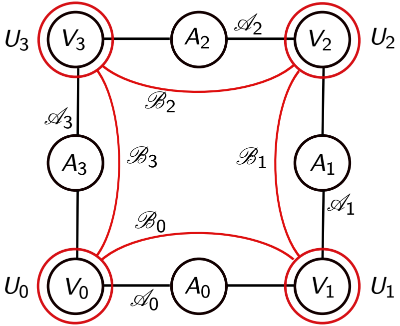

$W = \emptyset$

(3) There is a block $\mathscr{B}$ in $\mathscr{P}$ containing all nodal vertices of the 4-loop, i. e. $V_0 \cup V_1 \cup V_2 \cup V_3 \cup W \subset At(\mathscr{B})$.

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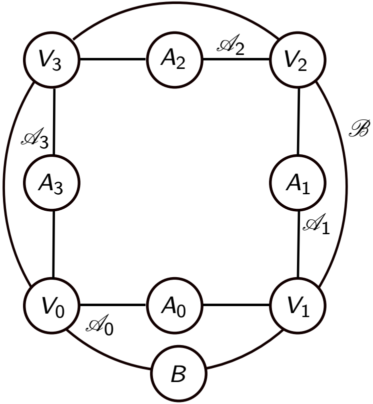

$W = \emptyset$

(4) There are two blocks $\mathscr{C}_0$ and $\mathscr{C}_1$ in $\mathscr{P}$ such that

$V_i \cup V_{i+1} \cup V_{i+2} \cup W \subset At(\mathscr{C}_0)$ and $V_{i+2} \cup V_{i+3} \cup V_{i+4} \cup W \subset At(\mathscr{C}_1)$,

for some $i \in \{0, 1, 2, 3\} \pmod 4$.

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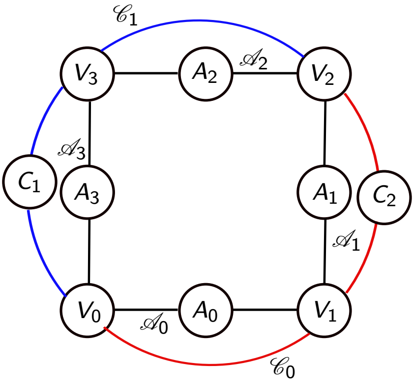
for some $i \in \{0, 1, 2, 3\} \pmod 4$.



$W = \emptyset$

Definition

Let $\mathscr{P}$ be an MV-algebra pasting containing a 4-loop $\mathscr{A}_0, \mathscr{A}_1, \mathscr{A}_2, \mathscr{A}_3$. We say that the 4-loop is **unbound** in $\mathscr{P}$ if none of the previous conditions $(1) - (4)$ is satisfied.

4-loops in MV-algebra pastings

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Theorem

An MV-algebra pasting $\mathscr{P}$ is a D-lattice if and only if $\mathscr{P}$ contains neither unbound 3-loops nor unbound 4-loops.

Distributive MV-algebra pastings

We know when an MV-algebra pasting is a D-lattice.

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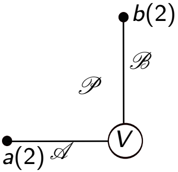
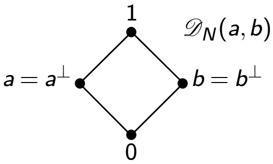
We give two cases of distributive MV-algebra pastings.

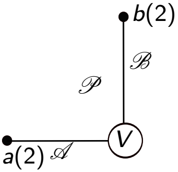
1. A pasting of two MV-algebras.

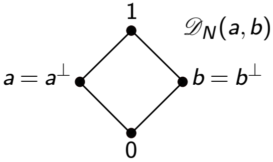
A pasting $\mathscr{P}$ of two MV-algebras $\mathscr{A}$ and $\mathscr{B}$ with a non-trivial center ($=V$ is nonempty set) is a distributive D-lattice if and only if the Greechie diagram of $\mathscr{P}$ is in the folowing form:





Distributive MV-algebra pastings





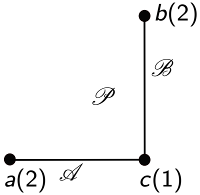
$V = At(\mathscr{A}) \cap At(\mathscr{B}) = \{c_1, c_2, \ldots, c_n\}$

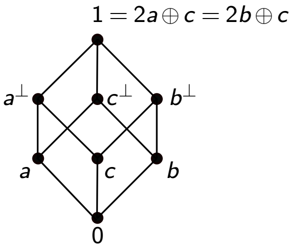
$\mathscr{D}_N(a,b)$ – the smallest non-compatble distributive difference lattice

$\mathscr{M}(\tau(c_1), \tau(c_2), \ldots, \tau(c_n))$ – MV-algebra generated by atoms of the set V

$\mathscr{P} \cong \mathscr{D}_N(a,b) \otimes \mathscr{M}(\tau(c_1), \tau(c_2), \ldots, \tau(c_n))$

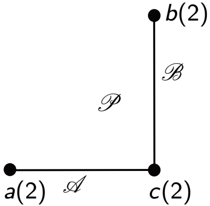
Distributive MV-algebra pastings

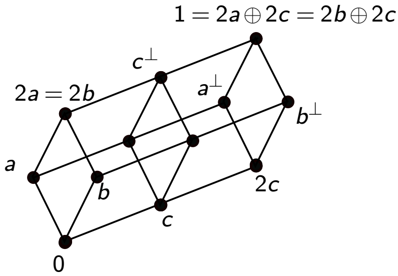




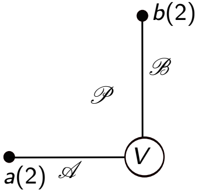
The Hasse diagram of this pasting is lattice-isomorphic to the power set of a set with three elements.

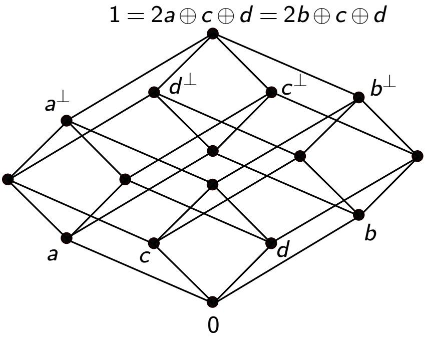
Distributive MV-algebra pastings





Distributive MV-algebra pastings



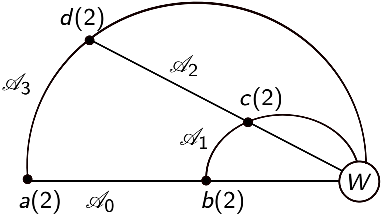


$$V = At(\mathscr{A}) \cap At(\mathscr{B}) = \{c, d\},\ \tau(c) = 1,\ \tau(d) = 1$$

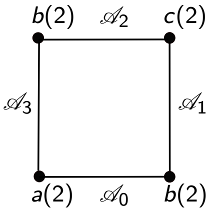
The Hasse diagram of this pasting is lattice-isomorphic to the power set of a set with four elements.

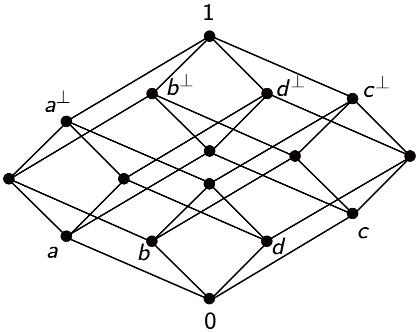
2. A pasting of four MV-algebras.

A pasting $\mathscr{P}$ of four MV-algebras is a distributive D-lattice if and only if the Greechie diagram of $\mathscr{P}$ is in the folowing form:

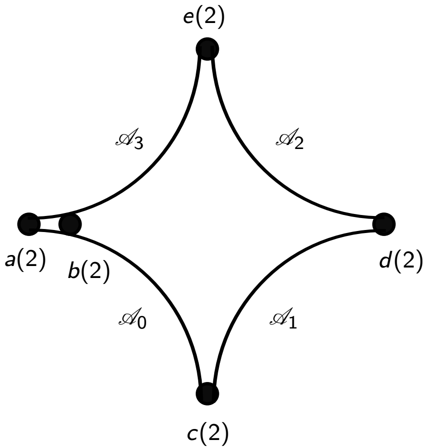


Distributive MV-algebra pastings





The Hasse diagram of this astroid is lattice-isomorphic to the power set of a set with four elements.



This astroid is a non-distributive D-lattice.

There are many other examples of distributive MV-algebra pastings, but I do not know yet, how their Greechie diagrams look.

THANK YOU FOR YOUR ATTENTION