Funkcie dvoch premenných

Užívateľské funkcie

$\mathrm{vec4sure}(v) := \left| \begin{array}{l} u \leftarrow (1) \\ v \cdot u \end{array} \right.$ vráti vektor 1×1 ak v je skalar, inak vráti pôvodný vektor v

$\mathrm{colvec4sure}(v) := \left| \begin{array}{l} u \leftarrow (1) \\ u \cdot v \end{array} \right.$ to isté, len pracuje s riadkovým vektorom

Funkcia 2 premenných

$$F(x,y) := \ln\left(x^2 + y^2\right)$$

Vyšetrovaný bod

$$\begin{pmatrix} x_0 & y_0 \end{pmatrix} := \begin{pmatrix} 1 & 1 \end{pmatrix}$$

Gradient funkcie

$$\mathrm{grad}(f,x,y) := \begin{pmatrix} \frac{d}{dx} f(x,y) & \frac{d}{dy} f(x,y) \end{pmatrix}^T$$

$$\mathrm{grad}(F,x,y)^T \rightarrow \begin{pmatrix} 2 \cdot \frac{x}{x^2 + y^2} & 2 \cdot \frac{y}{x^2 + y^2} \end{pmatrix} \qquad \mathrm{grad}\left(F, x_0, y_0\right) = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

Derivácia v smere

$$\mathrm{dvsmere}(f,x,y,v) := \left| \begin{array}{l} u \leftarrow \frac{v}{|v|} \\ \mathrm{grad}(f,x,y) \cdot u \end{array} \right.$$

$$\mathrm{dvsmere}\left[F, x_0, y_0, \begin{pmatrix} 8 \\ 6 \end{pmatrix}\right] = 1.4$$

$$\mathrm{dvsmere}\left[F, x_0, y_0, \begin{pmatrix} 8 \\ 6 \end{pmatrix}\right] \rightarrow \frac{7}{5}$$

Hessova matica

$$H(f,x,y) := \begin{pmatrix} \frac{d^2}{dx^2} f(x,y) & \frac{d}{dx}\frac{d}{dy} f(x,y) \\ \frac{d}{dy}\frac{d}{dx} f(x,y) & \frac{d^2}{dy^2} f(x,y) \end{pmatrix} \qquad H\left(F, x_0, y_0\right) = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$$

Diferenciál

$$\mathrm{Df}(f, x0, y0, x, y) := \left| \begin{array}{l} A \leftarrow \begin{pmatrix} x0 & y0 \end{pmatrix}^T \\ X \leftarrow \begin{pmatrix} x & y \end{pmatrix}^T \\ \begin{bmatrix} \mathrm{grad}(f, x0, y0)^T \cdot (X - A) \\ (X - A)^T \cdot H(f, x0, y0) \cdot (X - A) \end{bmatrix} \end{array} \right.$$

$$\mathrm{Df}\left(F, x_0, y_0, x, y\right) \rightarrow \begin{bmatrix} x - 2 + y \\ (-y + 1) \cdot (x - 1) + (-x + 1) \cdot (y - 1) \end{bmatrix}$$

Taylorov rozvoj
(stupňa "ord")

-známym vzorcom alebo
(ord = 2)

$$T\left(f, x_0, y_0, x, y, \mathrm{ord}\right) := f\left(x_0, y_0\right) + \sum_{k=1}^{\mathrm{ord}} \frac{1}{k!} \mathrm{Df}\left(f, x_0, y_0, x, y\right)_k$$

-zabudovanou funkciou
(stupen = 3 − 1)

$$T\left(F, x_0, y_0, x, y, 2\right) \text{ simplify } \rightarrow \ln(2) + 2 \cdot x - 3 + 2 \cdot y - y \cdot x$$

$$F(x,y) \left| \begin{array}{l} \text{series}, x = 1, y = 1, 3 \\ \text{simplify} \end{array} \right. \rightarrow \ln(2) + 2 \cdot x - 3 + 2 \cdot y - y \cdot x$$

(Dotyková rovina je Taylorov polynóm 1.stupňa)

$$F(x,y) := x^3 + 3x\cdot y + y^3$$

Voľné extrémy

stacionárne body

$$\text{Given} \quad \frac{d}{dx}F(x,y) = 0 \qquad \frac{d}{dy}F(x,y) = 0$$

$$\text{Find}(x,y) \rightarrow \begin{pmatrix} 0 & -1 & \frac{1}{2} + \frac{1}{2}\cdot i\cdot 3^{\frac{1}{2}} & \frac{1}{2} - \frac{1}{2}\cdot i\cdot 3^{\frac{1}{2}} \\ 0 & -1 & \frac{1}{2} - \frac{1}{2}\cdot i\cdot 3^{\frac{1}{2}} & \frac{1}{2} + \frac{1}{2}\cdot i\cdot 3^{\frac{1}{2}} \end{pmatrix}$$

kopírovať reálne korene! ---> $SB := \begin{pmatrix} 0 & -1 \\ 0 & -1 \end{pmatrix}$

rozhodovanie

```
volneX(f, SB) := | for i ∈ 1 .. cols(SB)
                 |   | (x  y) ← (SB^⟨i⟩)^T
                 |   | (M1  M2) ← ( H(f,x,y)_{1,1}   |H(f,x,y)| )
                 |   | v_{i,1} ← | if M2 > 0
                 |   |           |   | "maximum"  if M1 < 0
                 |   |           |   | "minimum"  if M1 > 0
                 |   |           | "sedlovy bod"  if M2 < 0
                 |   |           | "nevieme urcit"  otherwise
                 |   | v_{i,2} ← (x  y  f(x,y))
                 | v
```

$$\text{volneX}(F,SB) \rightarrow \begin{bmatrix} \text{"sedlovy bod"} & (0 \;\; 0 \;\; 0) \\ \text{"maximum"} & (-1 \;\; -1 \;\; 1) \end{bmatrix}$$

Viazané extrémy

Dosadzovacou metódou

$$F(x,y) := x\cdot y^2(4 - x - y)$$

hraničná krivka

$g(x,y) = 0$ --> $y(x)$

$$y_{hr}(x) := 6 - x$$

rozhodovanie

```
viazaneXD(f, fy) := | SB ← colvec4sure( root( d/dx f(x, fy(x)), x ) )^T
                    | for i ∈ 1 .. rows(SB)
                    |   | x ← SB_i
                    |   | y ← fy(x)
                    |   | dd ← d²/dx² f(x, fy(x))
                    |   | v_{i,1} ← | "maximum"  if dd < 0
                    |   |           | "minimum"  if dd > 0
                    |   |           | "nie je extrém"  otherwise
                    |   | v_{i,2} ← (x  y  f(x,y))
                    | v
```

$$\text{viazaneXD}(F, y_{hr}) \rightarrow \begin{bmatrix} \text{"minimum"} & (2 \;\; 4 \;\; -64) \\ \text{"maximum"} & (6 \;\; 0 \;\; 0) \end{bmatrix}$$

Lagrangeovou metódou

$$F(x,y) := y^2 - x$$

hraničná krivka
$g(x,y) = 0$

$$g_{hr}(x,y) := x^2 + y^2 - 1$$

Lagrangeova funkcia

$$\mathrm{Lagr}(f,g,x,y,\lambda) := f(x,y) - \lambda\cdot g(x,y)$$

$$\mathrm{Lagr}(F,g_{hr},x,y,\lambda) \rightarrow y^2 - x - \lambda\cdot(x^2 + y^2 - 1)$$

stacionárne body

$$\text{Given}\quad \frac{d}{dx}\mathrm{Lagr}(F,g_{hr},x,y,\lambda) = 0$$

$$\frac{d}{dy}\mathrm{Lagr}(F,g_{hr},x,y,\lambda) = 0$$

$$\frac{d}{d\lambda}\mathrm{Lagr}(F,g_{hr},x,y,\lambda) = 0 \qquad \mathrm{Find}(x,y,\lambda) \rightarrow \begin{pmatrix} 1 & -1 & \frac{-1}{2} & \frac{-1}{2} \\ 0 & 0 & \frac{1}{2}\cdot 3^{\frac{1}{2}} & \frac{-1}{2}\cdot 3^{\frac{1}{2}} \\ \frac{-1}{2} & \frac{1}{2} & 1 & 1 \end{pmatrix}$$

kopírovať reálne korene! ---> $SB := \begin{pmatrix} 1 & -1 & \frac{-1}{2} & \frac{-1}{2} \\ 0 & 0 & \frac{1}{2}\cdot 3^{\frac{1}{2}} & \frac{-1}{2}\cdot 3^{\frac{1}{2}} \\ \frac{-1}{2} & \frac{1}{2} & 1 & 1 \end{pmatrix}$

Hessova matica pre Lagrangeovu funkciu

$$H_\lambda(f,g,x,y,\lambda) := \begin{pmatrix} \frac{d^2}{dx^2}\mathrm{Lagr}(f,g,x,y,\lambda) & \frac{d}{dx}\frac{d}{dy}\mathrm{Lagr}(f,g,x,y,\lambda) \\ \frac{d}{dy}\frac{d}{dx}\mathrm{Lagr}(f,g,x,y,\lambda) & \frac{d^2}{dy^2}\mathrm{Lagr}(f,g,x,y,\lambda) \end{pmatrix} \qquad H_\lambda(F,g_{hr},x,y,\lambda) \rightarrow \begin{pmatrix} -2\cdot\lambda & 0 \\ 0 & 2 - 2\cdot\lambda \end{pmatrix}$$

rozhodovanie

```
viazaneXL(f,g,SB) := | for i ∈ 1 .. cols(SB)
                     |   | (x  y  λ) ← SB^⟨i⟩T
                     |   | (M1  M2) ← ( H_λ(f,g,x,y,λ)_1,1   |H_λ(f,g,x,y,λ)| )
                     |   | v_i,1 ← | if M2 > 0
                     |   |         |   | "maximum"  if M1 < 0
                     |   |         |   | "minimum"  if M1 > 0
                     |   |         | "sedlovy bod"  if M2 < 0
                     |   |         | "nevieme urcit"  otherwise
                     |   | v_i,2 ← (x  y  f(x,y))
                     | v
```

$$\mathrm{viazaneXL}(F,g_{hr},SB) \rightarrow \begin{bmatrix} \text{"minimum"} & (1\ \ 0\ \ -1) \\ \text{"sedlovy bod"} & (-1\ \ 0\ \ 1) \\ \text{"nevieme urcit"} & \begin{pmatrix} \frac{-1}{2} & \frac{1}{2}\cdot 3^{\frac{1}{2}} & \frac{5}{4} \end{pmatrix} \\ \text{"nevieme urcit"} & \begin{pmatrix} \frac{-1}{2} & \frac{-1}{2}\cdot 3^{\frac{1}{2}} & \frac{5}{4} \end{pmatrix} \end{bmatrix}$$