Diferencialna geometria plochy

```
(* rezervovane symboly: r,u,v,x,y,z, a,b,c,R, u0,v0,P0, ru,rv,ruu,
 ruv,rvv, ∇f, vn, fE,fF,fG,fL,fM,fN,fH,fK, hs,ppp, du,dv , assum*)

<< Calculus`VectorAnalysis`

r[u_, v_] = {a Cos[u] Cos[v], b Sin[u] Cos[v], c Sin[v]};
(* parametricka rovnica plochy *)
assum = {a > 0, b > 0, c > 0};
f[x_, y_, z_] = x^2/a^2 + y^2/b^2 + z^2/c^2 - R^2;(* implicitna rovnica plochy *)
u0 = 0; v0 = 0 ;
P0 = r[u0, v0];(* vyšetrovany bod *)

(* parcialne derivacie vektorovej funkcie r[u,v] podla parametrov u,v *)
ru[u_, v_] = D[r[u, v], u]; rv[u_, v_] = D[r[u, v], v];
ruu[u_, v_] = D[r[u, v], u, u]; ruv[u_, v_] = D[r[u, v], u, v]; rvv[u_, v_] = D[r[u, v], v,

{{ru[u, v], rv[u, v]}, {ru[u0, v0], rv[u0, v0]}} // Simplify // MatrixForm
```

$$\begin{pmatrix} \begin{pmatrix} -a\,Cos[v]\,Sin[u] \\ b\,Cos[u]\,Cos[v] \\ 0 \end{pmatrix} & \begin{pmatrix} -a\,Cos[u]\,Sin[v] \\ -b\,Sin[u]\,Sin[v] \\ c\,Cos[v] \end{pmatrix} \\ \begin{pmatrix} 0 \\ b \\ 0 \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \\ c \end{pmatrix} \end{pmatrix}$$

```
(* gradient plochy danej implicitne *)
∇f[x_, y_, z_] = Grad[f[x, y, z], Cartesian[x, y, z]]
```

$$\left\{\frac{2\,x}{a^2},\ \frac{2\,y}{b^2},\ \frac{2\,z}{c^2}\right\}$$

```
X = {x, y, z};(* vseobecny bod *)
```

Dotykova rovina a normala ku ploche

```
vn[u_, v_] = ru[u, v] × rv[u, v];
vn[##] & @@@ {{u, v}, {u0, v0}} // Simplify // TableForm
```

$b\,c\,Cos[u]\,Cos[v]^2$	$a\,c\,Cos[v]^2\,Sin[u]$	$a\,b\,Cos[v]\,Sin[v]$
$b\,c$	0	0

```
dotykovarovinaV[u_, v_] = (X - P0).vn[u, v];
normalaV[u_, v_] = P0 + λ vn[u, v];

dotykovarovinaI[x0_, y0_, z0_] = (X - {x0, y0, z0}).∇f[x0, y0, z0];
normalaI[x0_, y0_, z0_] = {x0, y0, z0} + λ ∇f[x0, y0, z0];

{dotykovarovinaV[##], normalaV[##]} & @@@ {{u, v}, {u0, v0}} // Simplify // TableForm
```

$Cos[v]\,(b\,c\,(-a+x)\,Cos[u]\,Cos[v]+a\,c\,y\,Cos[v]\,Sin[u]+a\,b\,z\,Sin[v])$	$a+b\,c\,\lambda\,Co$ $a\,c\,\lambda\,Cos[v]^2\,Sin[$ $a\,b\,\lambda\,Cos[v]\,Sin[v$
$b\,c\,(-a+x)$	$a+b\,c\,\lambda$ 0 0

```
{dotykovarovinaI[##], normalaI[##]} &@@ P0 // Simplify // TableForm
```

$-2 + \frac{2x}{a}$

$a + \frac{2\lambda}{a}$ 0 0

Krivosti

```
dlvec[v_] := Sqrt[v.v]

vn[u_, v_] = vn[u, v] / dlvec[vn[u, v]] ;(* normovanie vektora normály *)

fE[u_, v_] = ru[u, v].ru[u, v]; fF[u_, v_] = ru[u, v].rv[u, v]; fG[u_, v_] = rv[u, v].rv[u
fL[u_, v_] = vn[u, v].ruu[u, v];
fM[u_, v_] = vn[u, v].ruv[u, v]; fN[u_, v_] = vn[u, v].rvv[u, v];

zakladneformy[u_, v_] = {fE[u, v] du^2 + 2 fF[u, v] du dv + fG[u, v] dv^2,
   fL[u, v] du^2 + 2 fM[u, v] du dv + fN[u, v] dv^2};
normalovakrivost[u_, v_] = %[[2]] / %[[1]];

Simplify[{zakladneformy[u0, v0], normalovakrivost[u0, v0]}, assum] // MatrixForm
```

$$\begin{pmatrix} \{b^2\,du^2 + c^2\,dv^2,\ -a\,(du^2+dv^2)\} \\ -\frac{a\,(du^2+dv^2)}{b^2\,du^2+c^2\,dv^2} \end{pmatrix}$$

$$fH[u\_, v\_] = \frac{fE[u, v]\, fN[u, v] + fL[u, v]\, fG[u, v] - 2\, fF[u, v]\, fM[u, v]}{2\,(fE[u, v]\, fG[u, v] - fF[u, v]\verb|^|2)};$$

$$fK[u\_, v\_] = \frac{fL[u, v]\, fN[u, v] - fM[u, v]\verb|^|2}{fE[u, v]\, fG[u, v] - fF[u, v]\verb|^|2};$$

$$hlavnekrivosti[u\_, v\_] = fH[u, v] + \{1, -1\}\sqrt{fH[u, v]\verb|^|2 - fK[u, v]};$$

```
(hk := hlavnekrivosti[u, v];
 uplnakrivost[u_, v_] = hk[[1]] hk[[2]];
 strednakrivost[u_, v_] = (hk[[1]] + hk[[2]]) / 2;
 Clear[hk];)

hlavnekrivosti[u0, v0] // Simplify
```

$$\left\{\frac{b\,c\left(b\,c\sqrt{b^2\,c^2}\sqrt{\frac{a^2\,(b^2-c^2)^2}{b^4\,c^4}} - a\,(b^2+c^2)\right)}{2\,(b^2\,c^2)^{3/2}},\ \frac{b\,c\left(-b\,c\sqrt{b^2\,c^2}\sqrt{\frac{a^2\,(b^2-c^2)^2}{b^4\,c^4}} - a\,(b^2+c^2)\right)}{2\,(b^2\,c^2)^{3/2}}\right\}$$

```
Simplify[hlavnekrivosti[u0, v0], {a > 0, b > 0, c > 0, b > c}]
```

$$\left\{-\frac{a}{b^2},\ -\frac{a}{c^2}\right\}$$

```
{uplnakrivost[u0, v0], strednakrivost[u0, v0]} // Simplify
```

$$\left\{\frac{a^2}{b^2\,c^2},\ -\frac{a\,b\,c\,(b^2+c^2)}{2\,(b^2\,c^2)^{3/2}}\right\}$$

Hlavne smery

```
hs[u_, v_] = (fL[u, v] fF[u, v] - fM[u, v] fE[u, v]) du^2 +
   (fL[u, v] fG[u, v] - fN [u, v] fE[u, v]) du dv + (fM[u, v] fG[u, v] - fN[u, v] fF[u, v]) dv'
ppp[u_, v_] = dlvec[ru[u, v] du + rv[u, v] dv];
```

```
solution = Solve[{hs[u0, v0] == 0, ppp[u0, v0] == 1}, {du, dv}]
```

Solve::ifun : Inverse functions are being used by Solve, so some solutions may not be found.

$$\left\{\left\{du \to -\frac{1}{b},\ dv \to 0\right\},\ \left\{du \to \frac{1}{b},\ dv \to 0\right\},\ \left\{dv \to -\frac{1}{c},\ du \to 0\right\},\ \left\{dv \to \frac{1}{c},\ du \to 0\right\}\right\}$$

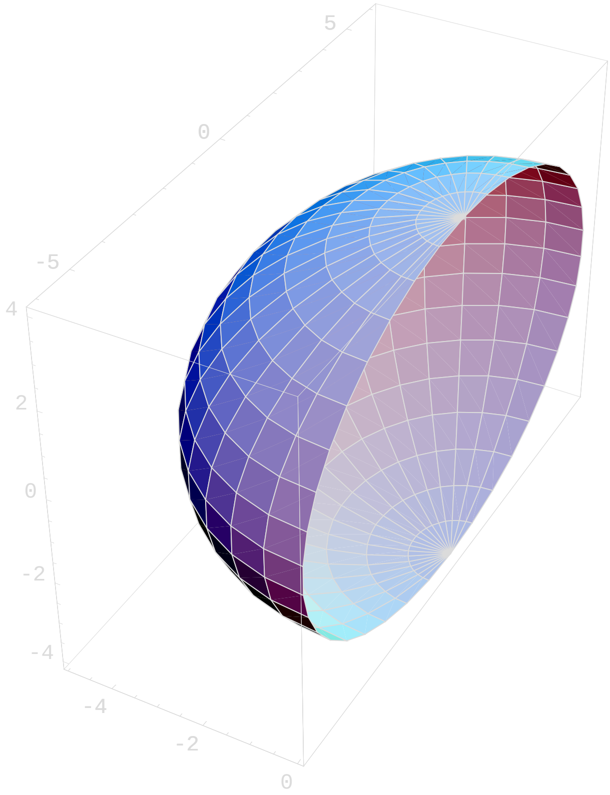
```
ru[u0, v0] du + rv[u0, v0] dv /. solution
RowReduce[%](* pre pripad, ze najde linearne zavisle vektory *)
```

{{0, -1, 0}, {0, 1, 0}, {0, 0, -1}, {0, 0, 1}}

{{0, 1, 0}, {0, 0, 1}, {0, 0, 0}, {0, 0, 0}}

Graficke znazornenie

```
(a = 5; b = 6; c = 4;)
ParametricPlot3D[r[u, v], {u, π / 2, 3 π / 2}, {v, -π / 2, π / 2}]
Clear[a, b, c]
```



- Graphics3D -