Diferencialna geometria krivky

```
In[1]:=  (* rezervovane symboly: P,P1,P2,P3, t0,t, vd,vb,vn, λ, A,x,y,z, s *)
```

```
In[11]:= P[t_] = {r t Cos[Log[t]], r t Sin[Log[t]], b t}; (* vektorova rovnica krivky *)
         t0 = 1; (* vysetrovany bod *)
```

```
In[13]:= (* parcialne derivacie vektorovej funkcie P[t] podla parametra t *)
         P1[t_] = D[P[t], t];
         P2[t_] = D[P[t], t, t];
         P3[t_] = D[P[t], t, t, t];
```

```
In[7]:=  {{P1[t], P2[t], P3[t]}, {P1[t0], P2[t0], P3[t0]}} // Simplify // MatrixForm
```

Out[7]//MatrixForm=

$$\begin{pmatrix} \begin{pmatrix} r\,(\mathrm{Cos}[\mathrm{Log}[t]] - \mathrm{Sin}[\mathrm{Log}[t]]) \\ r\,(\mathrm{Cos}[\mathrm{Log}[t]] + \mathrm{Sin}[\mathrm{Log}[t]]) \\ b \end{pmatrix} & \begin{pmatrix} -\frac{r\,(\mathrm{Cos}[\mathrm{Log}[t]]+\mathrm{Sin}[\mathrm{Log}[t]])}{t} \\ \frac{r\,(\mathrm{Cos}[\mathrm{Log}[t]]-\mathrm{Sin}[\mathrm{Log}[t]])}{t} \\ 0 \end{pmatrix} & \begin{pmatrix} \frac{2\,r\,\mathrm{Sin}[\mathrm{Log}[t]]}{t^2} \\ -\frac{2\,r\,\mathrm{Cos}[\mathrm{Log}[t]]}{t^2} \\ 0 \end{pmatrix} \\ \begin{pmatrix} r \\ r \\ b \end{pmatrix} & \begin{pmatrix} -r \\ r \\ 0 \end{pmatrix} & \begin{pmatrix} 0 \\ -2\,r \\ 0 \end{pmatrix} \end{pmatrix}$$

Sprievodny trojhran

```
In[16]:= vd[t_] = P1[t];
         vb[t_] = P1[t] × P2[t];
         vn[t_] = (P1[t] × P2[t]) × P1[t];
```

```
In[11]:= {vd[#], vb[#], vn[#]} & /@ {t, t0} // Simplify // MatrixForm
```

Out[11]//MatrixForm=

$$\begin{pmatrix} \begin{pmatrix} r\,(\mathrm{Cos}[\mathrm{Log}[t]] - \mathrm{Sin}[\mathrm{Log}[t]]) \\ r\,(\mathrm{Cos}[\mathrm{Log}[t]] + \mathrm{Sin}[\mathrm{Log}[t]]) \\ b \end{pmatrix} & \begin{pmatrix} -\frac{b\,r\,(\mathrm{Cos}[\mathrm{Log}[t]]-\mathrm{Sin}[\mathrm{Log}[t]])}{t} \\ -\frac{b\,r\,(\mathrm{Cos}[\mathrm{Log}[t]]+\mathrm{Sin}[\mathrm{Log}[t]])}{t} \\ \frac{2\,r^2}{t} \end{pmatrix} & \begin{pmatrix} -\frac{r\,(b^2+2\,r^2)\,(\mathrm{Cos}[\mathrm{Log}[t]]+\mathrm{Sin}[\mathrm{Log}}{t} \\ \frac{r\,(b^2+2\,r^2)\,(\mathrm{Cos}[\mathrm{Log}[t]]-\mathrm{Sin}[\mathrm{Log}}{t} \\ 0 \end{pmatrix} \\ \begin{pmatrix} r \\ r \\ b \end{pmatrix} & \begin{pmatrix} -b\,r \\ -b\,r \\ 2\,r^2 \end{pmatrix} & \begin{pmatrix} -r\,(b^2 + 2\,r^2) \\ r\,(b^2 + 2\,r^2) \\ 0 \end{pmatrix} \end{pmatrix}$$

```
In[19]:= dotycnica[t_] = P[t] + λ vd[t];
         binormala[t_] = P[t] + λ vb[t];
         hlavnanormala[t_] = P[t] + λ vn[t];
```

```
In[22]:= {dotycnica[t0], binormala[t0], hlavnanormala[t0]} // Transpose // TableForm
```

Out[22]//TableForm=

$r + r\,\lambda$	$r - b\,r\,\lambda$	$r + (-b^2\,r - 2\,r^3)\,\lambda$
$r\,\lambda$	$-b\,r\,\lambda$	$(b^2\,r + 2\,r^3)\,\lambda$
$b + b\,\lambda$	$b + 2\,r^2\,\lambda$	b

```
In[23]:= A = {x, y, z};
         oskulacna[t_] = (A - P[t]).vb[t];
         normalova[t_] = (A - P[t]).vd[t];
         rektifikacna[t_] = (A - P[t]).vn[t];
```

In[27]:= {oskulacna[t0], normalova[t0], rektifikacna[t0]} // TableForm

Out[27]//TableForm=

$$-b\,r\,(-r+x)-b\,r\,y+2\,r^2\,(-b+z)$$
$$r\,(-r+x)+r\,y+b\,(-b+z)$$
$$(-b^2\,r-2\,r^3)\,(-r+x)+(b^2\,r+2\,r^3)\,y$$

Krivosti

```
In[28]:= dlvec[v_] := Sqrt[v.v]
```

In[29]:= $\text{flexia}[t\_] = \sqrt{\dfrac{\text{dlvec}[\text{vb}[t]]\text{^}2}{\text{dlvec}[\text{P1}[t]]\text{^}6}};$

$\text{torzia}[t\_] = \dfrac{\text{vb}[t].\text{P3}[t]}{\text{dlvec}[\text{vb}[t]]\text{^}2};$

```
In[31]:= {flexia[t0], torzia[t0]} // Simplify // TableForm
```

Out[31]//TableForm=

$$\sqrt{2}\,\sqrt{\frac{r^2}{(b^2+2\,r^2)^2}}$$
$$\frac{b}{b^2+2\,r^2}$$

Prirodzena parametrizacia a Dlzka krivky

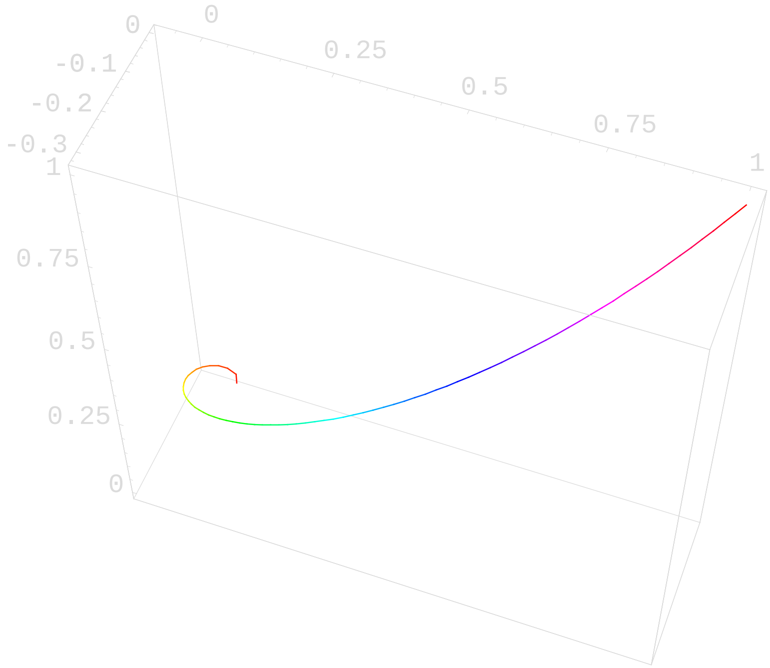
```
In[32]:= s[t_, t1_, t2_] = Integrate[Sqrt[P1[t].P1[t]], {t, t1, t2}];

In[33]:= s[t, 0, t]
         (* prirodzene parametricke vyjadrenie
           dostavame ak sa "t" da vyjadrit ako funkcia dlzky "s" *)
```

Out[33]= $\sqrt{b^2+2\,r^2}\;t$

Graficke znazornenie

In[55]:=
```
(r = 1; b = 1;)
ParametricPlot3D[Append[P[t], Hue[t]],
 {t, 0.0001, 1}, ViewPoint -> {0.734, -1.995, 2.926}]
Clear[r, b];
```



Out[56]= - Graphics3D -