Diferenciálna geometria krivky

Užívateľské funkcie

derivácia vektorovej funkcie:

$$Dv(f,t,ord) := \left| \begin{array}{l} n \leftarrow length(f(t)) \\ \text{for } i \in 1..n \\ \quad vec_i \leftarrow \dfrac{d^{ord}}{dt^{ord}}(f(t))_i \\ vec \end{array} \right.$$

Vyšetrovaný bod:

$$t_0 := 0$$

Vektorová rovnica krivky a jej derivácie:

$$P(t) := \begin{pmatrix} \cos(t) & \sin(t) & t^2 \end{pmatrix}^T \qquad P(t_0)^T = (1 \quad 0 \quad 0)$$

$$P'(t) := Dv(P,t,1) \qquad P'(t)^T \rightarrow (-\sin(t) \quad \cos(t) \quad 2\cdot t) \qquad P'(t_0)^T = (0 \quad 1 \quad 0)$$

$$P''(t) := Dv(P,t,2) \qquad P''(t)^T \rightarrow (-\cos(t) \quad -\sin(t) \quad 2) \qquad P''(t_0)^T = (-1 \quad 0 \quad 2)$$

$$P'''(t) := Dv(P,t,3) \qquad P'''(t)^T \rightarrow (\sin(t) \quad -\cos(t) \quad 0) \qquad P'''(t_0)^T = (0 \quad -1 \quad 0)$$

Sprievodný trojhran

dotyčnica:

$$vek_d(t) := P'(t) \qquad vek_d(t_0)^T = (0 \quad 1 \quad 0)$$

$$priam_d(t,\lambda) := P(t) + \lambda\cdot vek_d(t) \qquad priam_d(t_0,\lambda)^T \rightarrow (1 \quad \lambda \quad 0)$$

binormála:

$$vek_b(t) := P'(t) \times P''(t) \qquad vek_b(t_0)^T = (2 \quad 0 \quad 1)$$

$$priam_b(t,\lambda) := P(t) + \lambda\cdot vek_b(t) \qquad priam_b(t_0,\lambda)^T \rightarrow (1 + 2\cdot\lambda \quad 0 \quad \lambda)$$

hlavná normála:

$$vek_n(t) := vek_b(t) \times vek_d(t) \qquad vek_n(t_0)^T = (-1 \quad 0 \quad 2)$$

$$priam_n(t,\lambda) := P(t) + \lambda\cdot vek_n(t) \qquad priam_n(t_0,\lambda)^T \rightarrow (1 - \lambda \quad 0 \quad 2\cdot\lambda)$$

oskulačná rovina:

$$rov_b(t,x,y,z) := \left((x \quad y \quad z)^T - P(t)\right)\cdot vek_b(t)$$

$$rov_b(t_0,x,y,z) \rightarrow 2\cdot x - 2 + z$$

normálová rovina:

$$rov_d(t,x,y,z) := \left((x \quad y \quad z)^T - P(t)\right)\cdot vek_d(t)$$

$$rov_d(t_0,x,y,z) \rightarrow y$$

rektifikačná rovina:

$$rov_n(t,x,y,z) := \left((x \quad y \quad z)^T - P(t)\right)\cdot vek_n(t)$$

$$rov_n(t_0,x,y,z) \rightarrow -x + 1 + 2\cdot z$$

Krivosti

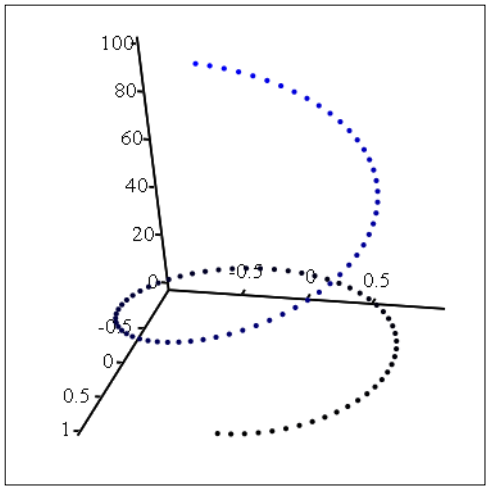
prvá krivosť (flexia):

$$flexia(t) := \frac{\left|vek_b(t)\right|}{\left(\left|vek_d(t)\right|\right)^3} \qquad flexia(t_0) = 2.236 \qquad flexia(t_0) \rightarrow 5^{\frac{1}{2}}$$

druhá krivosť (torzia):

$$torzia(t) := \frac{vek_b(t)\cdot P'''(t)}{\left(\left|vek_b(t)\right|\right)^2} \qquad torzia(t_0) = 0 \qquad torzia(t_0) \rightarrow 0$$

Graf priestorovej krivky:



P