Modeling point's position time series with respect to common trend

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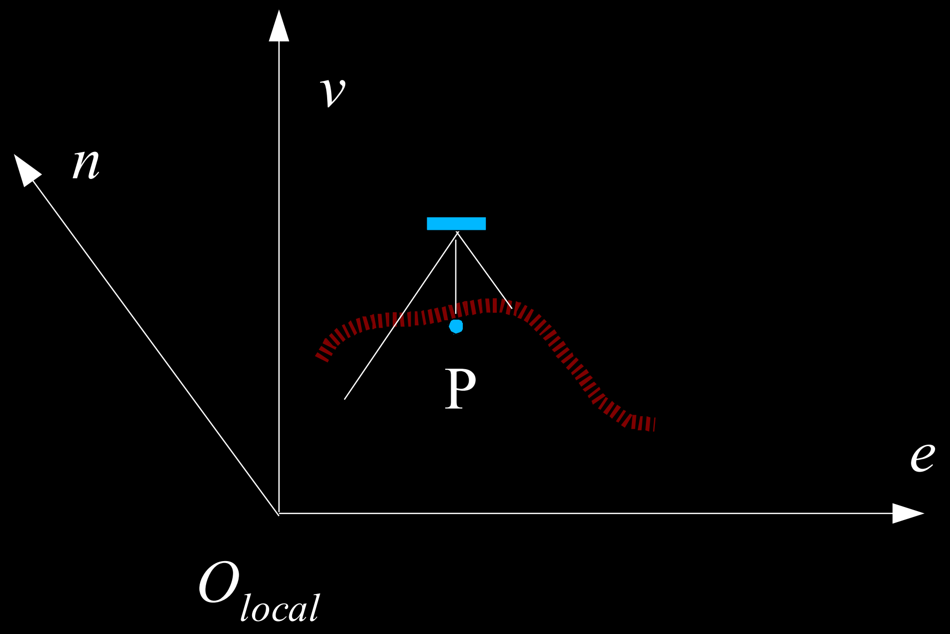
- Objective: To compare 3 ways of processing data
- Primary field of application: Geodesy
- Idea proposed within:

Komorníkova and Komorník,
Time series models for earth's crust kinematics, 2002

We've got a point having its coordinates in horizontal coordinate system (*n, e, v*).

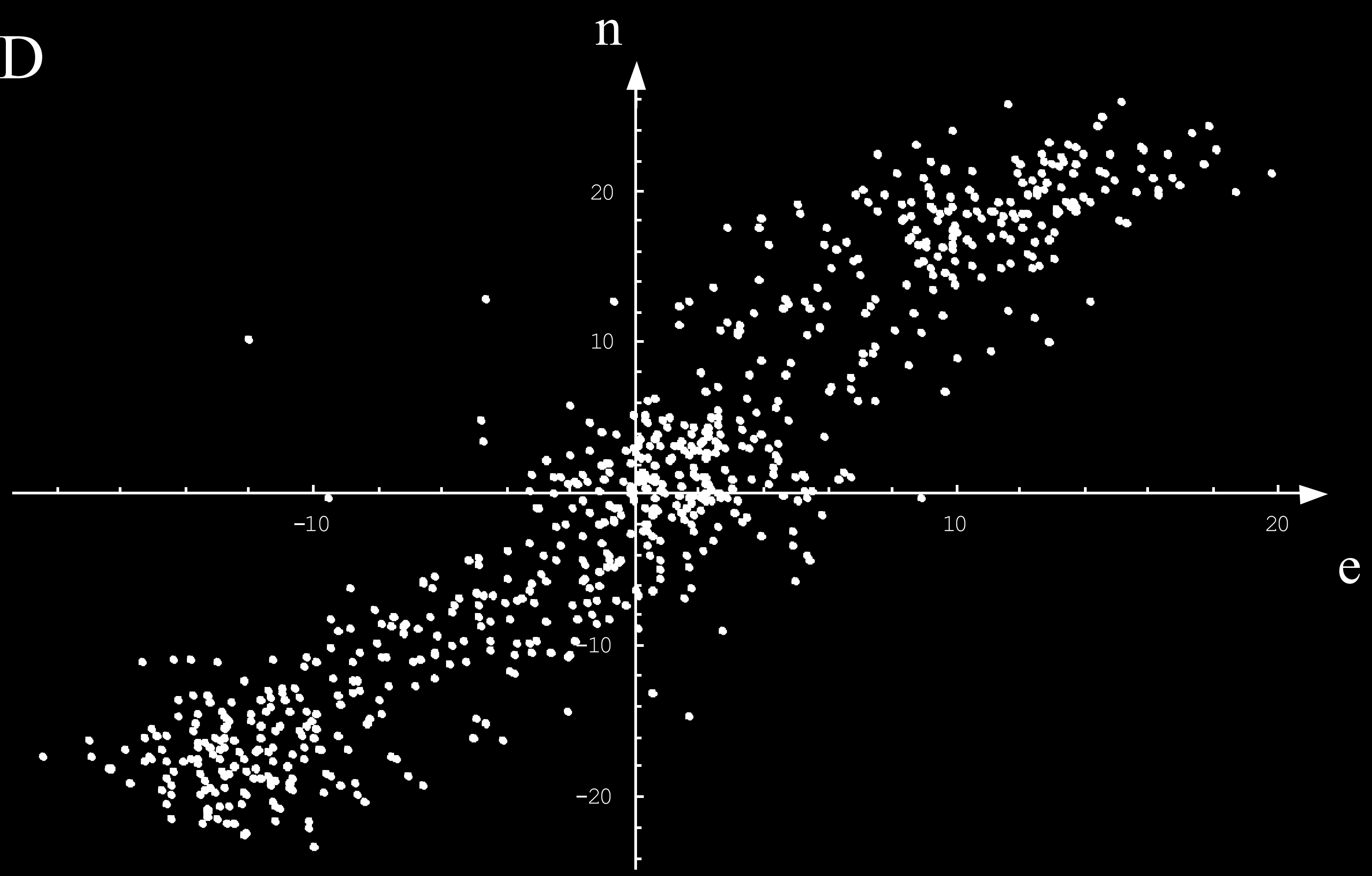
Components:

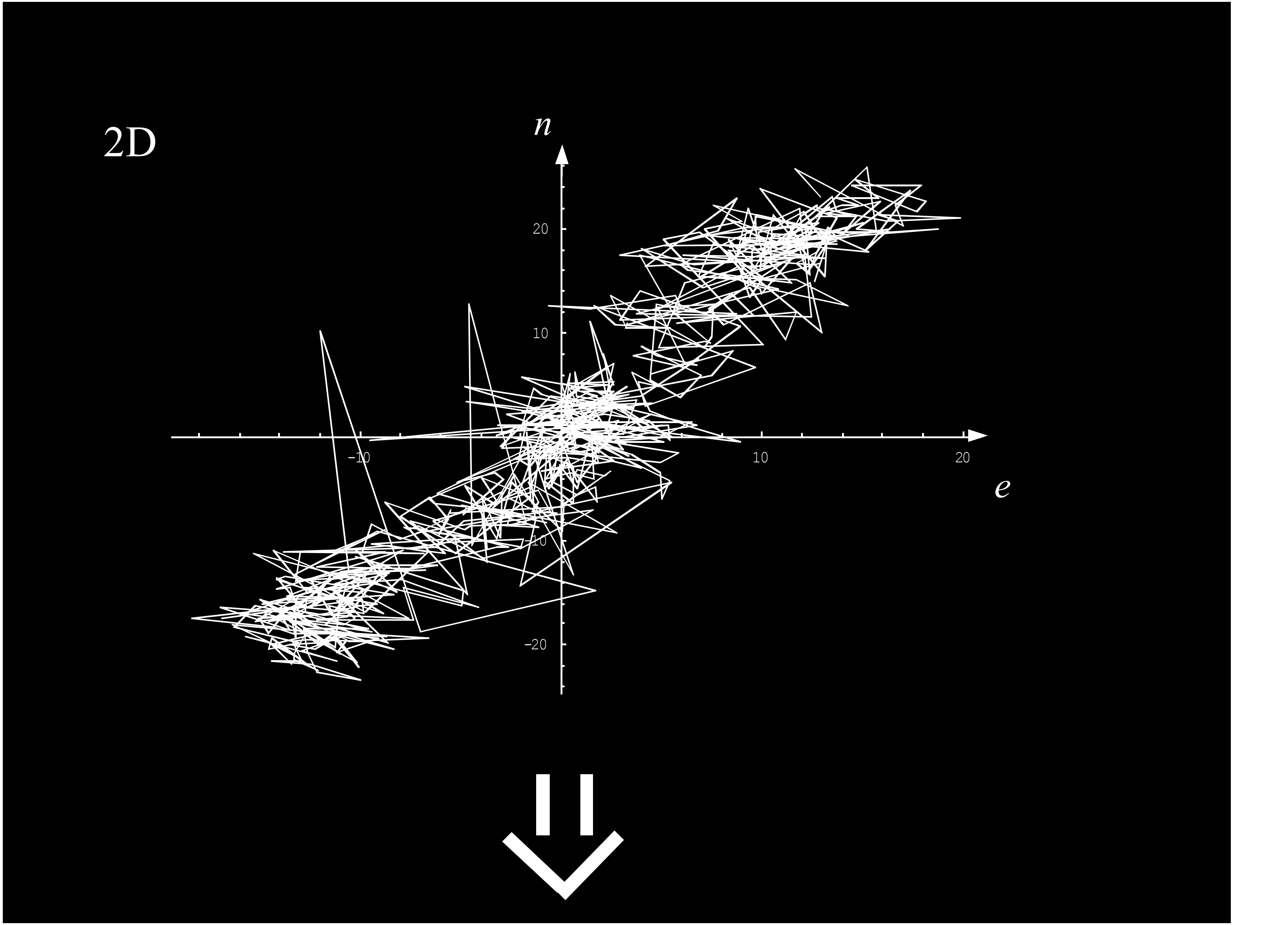
n – north
e – east
v - vertical

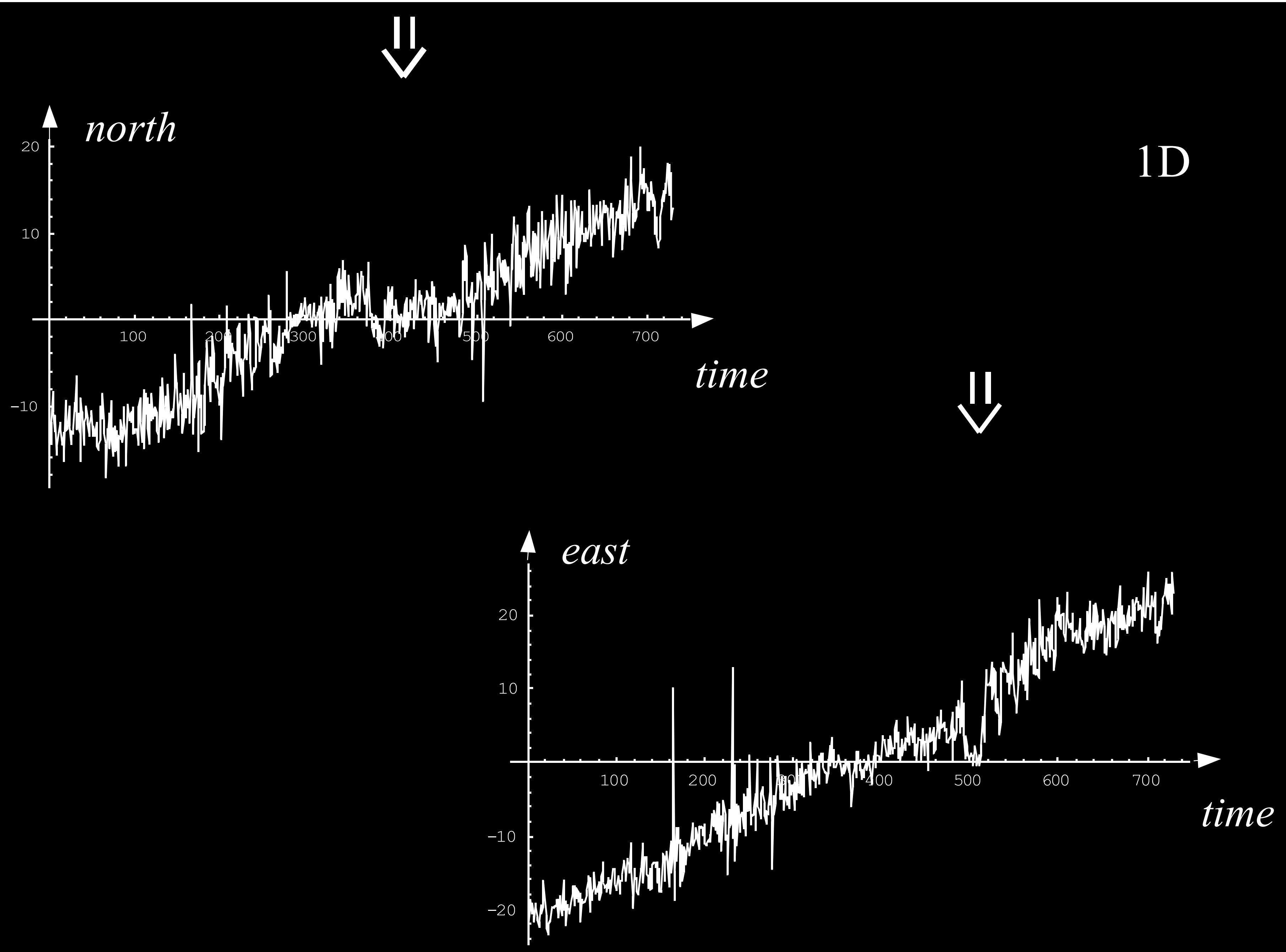


The position has been observed by means of Global Positioning System (GPS) and given daily for 2 years. So we have 730 time points per every coordinate.

2D

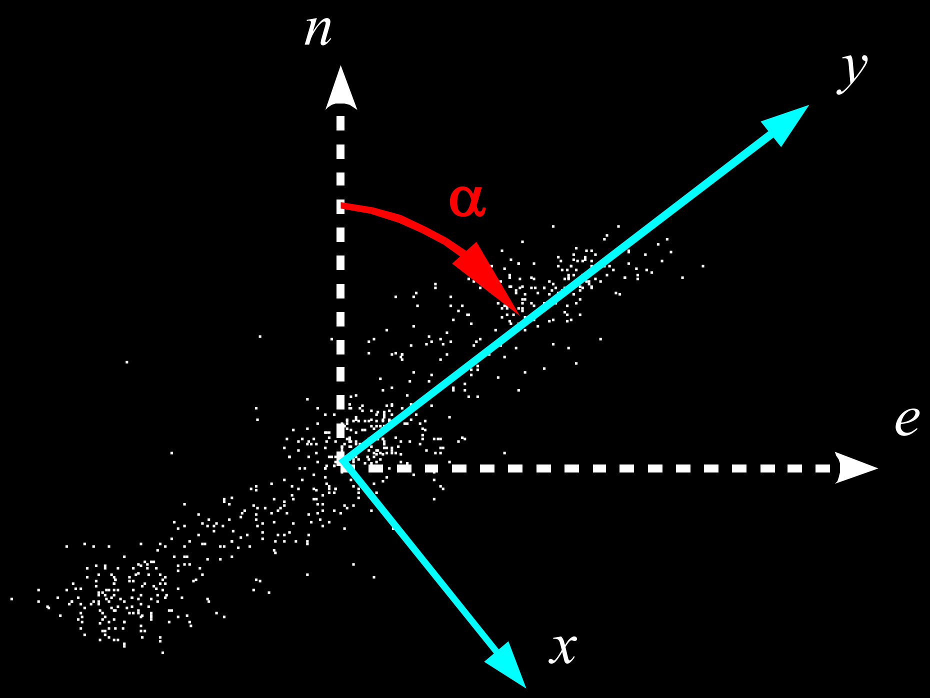



2D
n
20
10
-10
10
20
e
-10
-20


north
20
10
-10
100
200
300
400
500
600
700
time
1D
east
20
10
-10
-20
100
200
300
400
500
600
700
time

3 ways to deal with such a geodetic data:

- Process it separately as 2 univariate time series
- Accept the interrelationship and use the multivariate modeling methods
- Think about physical and geometrical background and <u>transform the data with respect to common trend</u>



$$\begin{pmatrix} y \\ x \end{pmatrix} = \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} n \\ e \end{pmatrix}$$

$\alpha = ?$

Rotation α of coordinate system (*n, e*) into common trend direction

1.) deterministic trend:

$$n = a_1 + b_1 t$$
$$e = a_2 + b_2 t$$

t - time
a, b - regr. parameters

substitution: *n(t), e(t)* → *x(n,e)*

$$x = -(a_1 + b_1\ t)\cos\alpha + (a_2 + b_2\ t)\sin\alpha$$
$$x = (a_2\cos\alpha - a_1\sin\alpha) + \underbrace{(b_2\cos\alpha - b_1\sin\alpha)}_{0}\,t$$

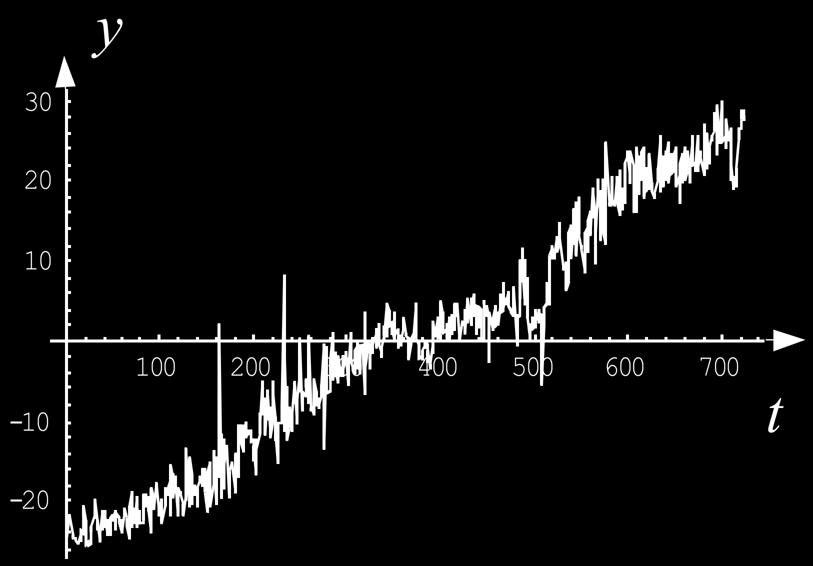
$$\alpha = \arctan\left(\frac{b_2}{b_1}\right)$$

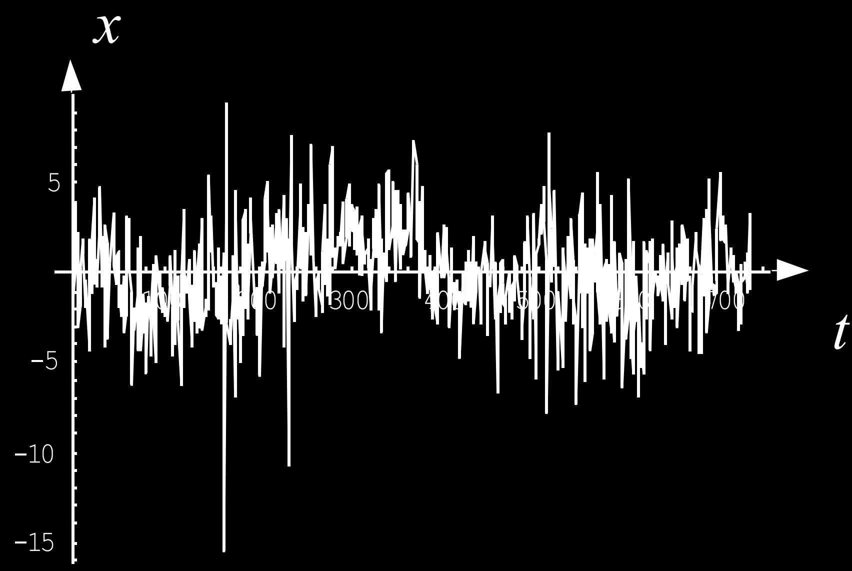
or

$$\alpha = \arctan(b_0)$$

2.) stochastic trend:

$$e_t = a_0 + b_0\ n_t$$

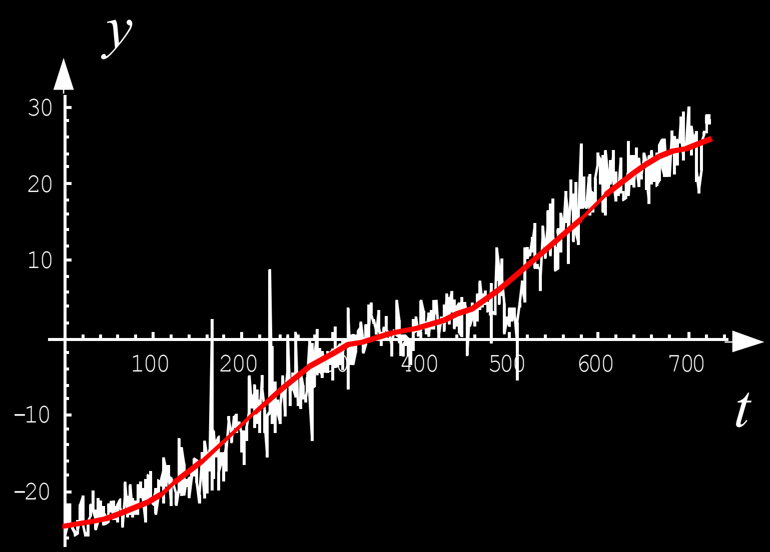


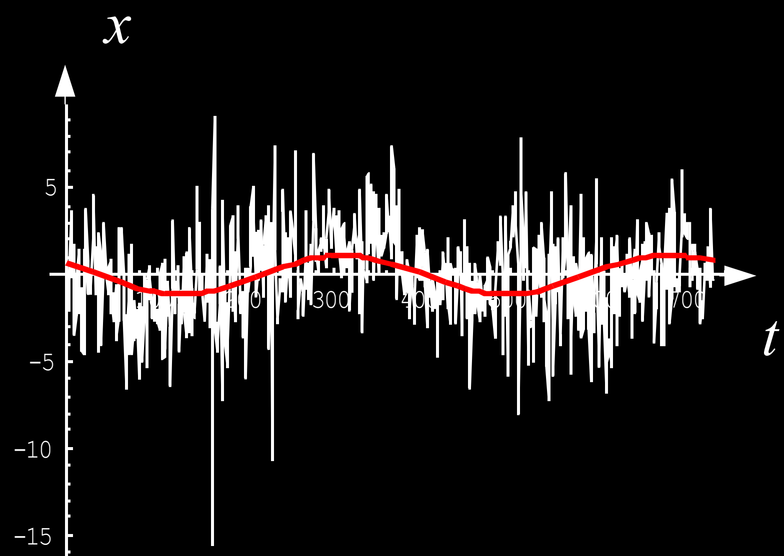


Next step is to decompose y, *x* into:

- linear trend
- seasonal component
- residuals →

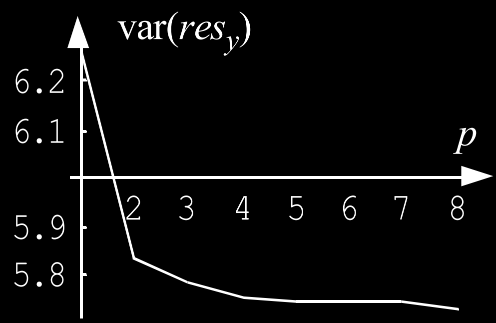
and to model the residuals with AR(p)

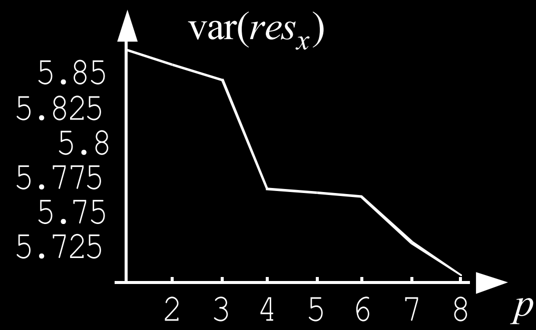




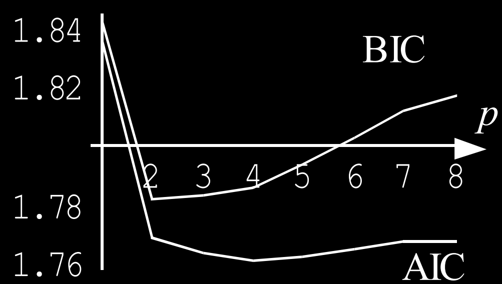
The order p of autoregressive model AR(p) is chosen by employing:

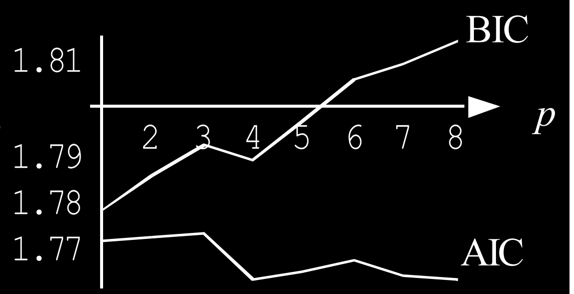
- plot of residuals' variances





- information criteria



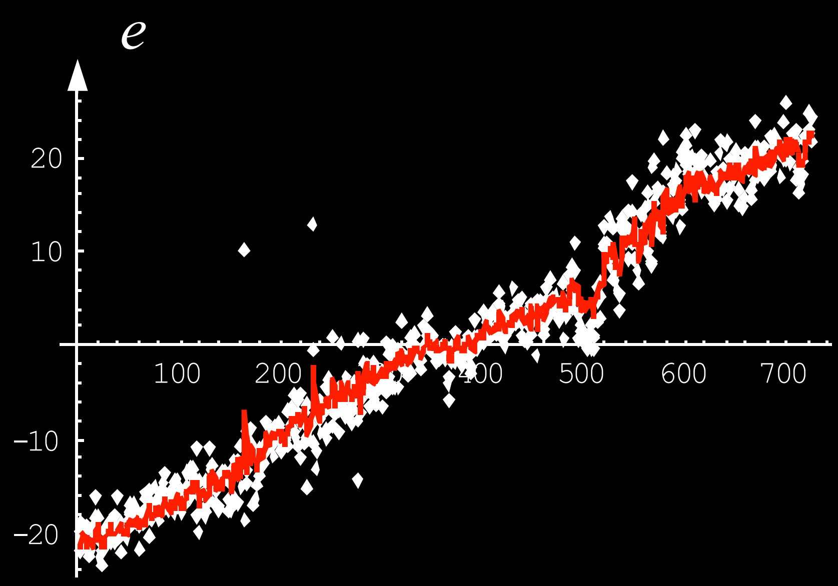


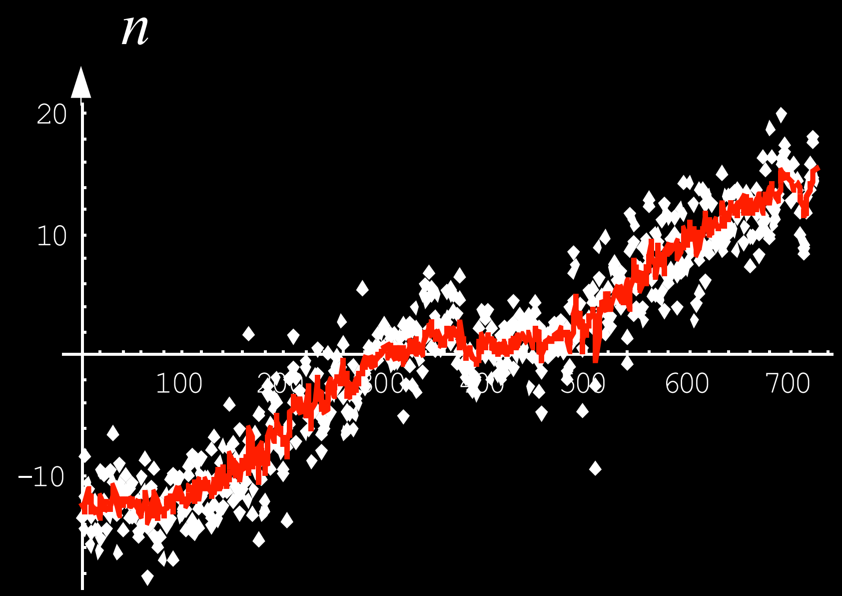
So the whole model for *y, x* has the form

$$y_{\mathrm{m}}\,,\; x_{\mathrm{m}} = \text{ trend } + \text{seasonal component} + \text{ARmodel}\,(p)$$

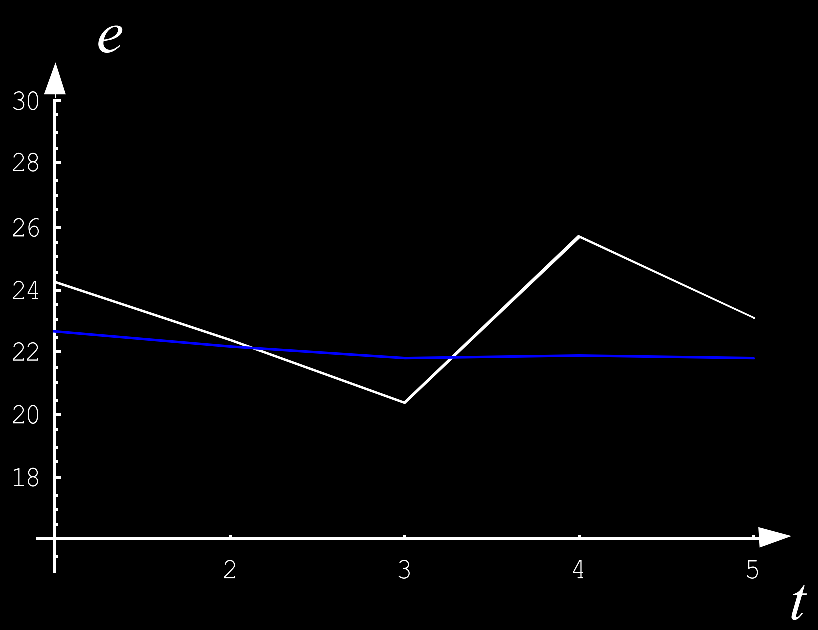
Because transformation matrix $M_{n,e\rightarrow y,x}$ is orthogonal, reverse transformation is provided simply by transposed M:

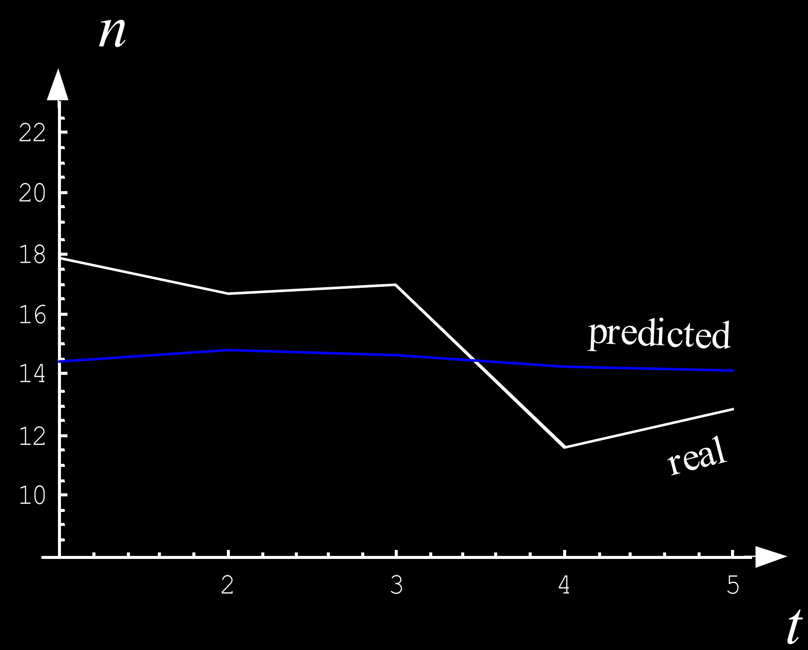
$$\begin{pmatrix} n_m \\ e_m \end{pmatrix} = \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} y_m \\ x_m \end{pmatrix}$$





Prediction





and some of its efficiency masures:

- mean square error
- mean percentage error

$$mse = \frac{1}{k} \sum_{t=1}^{k} (real_t - model_t)^2$$

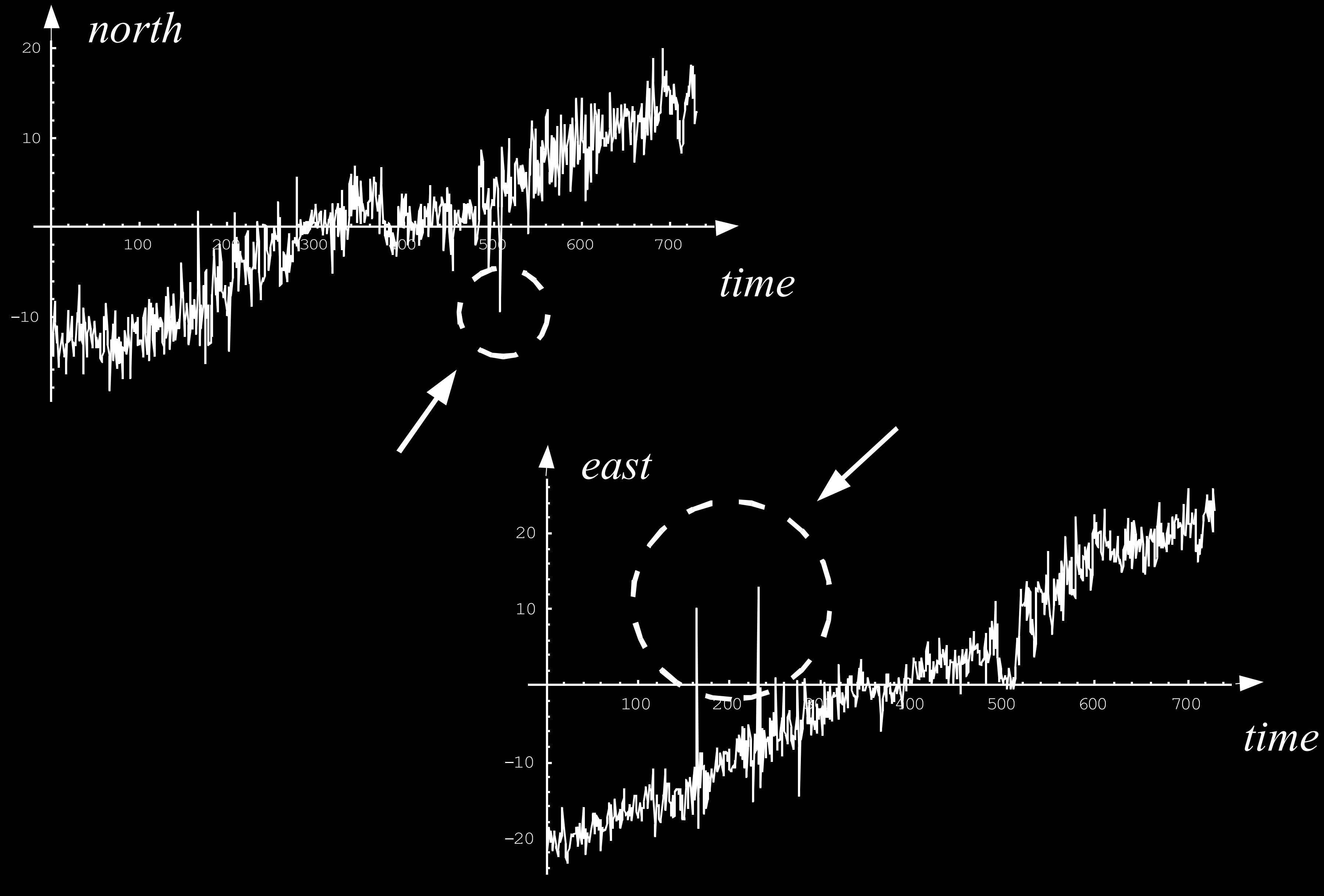
$$mpe = \frac{1}{k} \sum_{t=1}^{k} \frac{real_t - model_t}{real_t} 100\,\%$$

Results

- trend n 13.2 mm / year, e 21.5 mm / year } common trend y 25.2 mm / year
- seasonality
  - n amplitude = 2.2 mm, period = 365days
  - e 1.6 mm
  - y 2.5 mm
  - x 1.1 mm
-

method	variable	order p	mse [mm^2]	mpe [%]
1.) independent univariate time series	n	1	7.40	5.08
	e	4	3.70	2.88
2.) multivariate time series	n	2	8.13	5.44
	e	2	5.06	5.13
3.) respecting common trend	n	2 (y)	5.90	4.04
	e	4 (x)	4.10	2.49

removing outliers



Results

method	variable	order p	mse [mm^2]	mpe [%]
1.) independent univariate time series	n	1	7.37	4.94
	e	4	3.34	1.85
2.) multivariate time series	n	4	7.52	5.01
	e	4	4.23	4.20
3.) respecting common trend	n	4 (y)	**5.59**	**1.97**
	e	4 (x)	**3.33**	**0.50**

Thank you!

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