Modelling point's position time series in the light of cointegration

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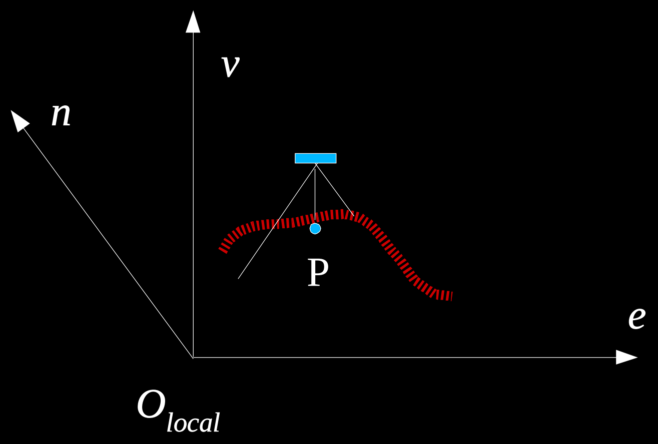
Data

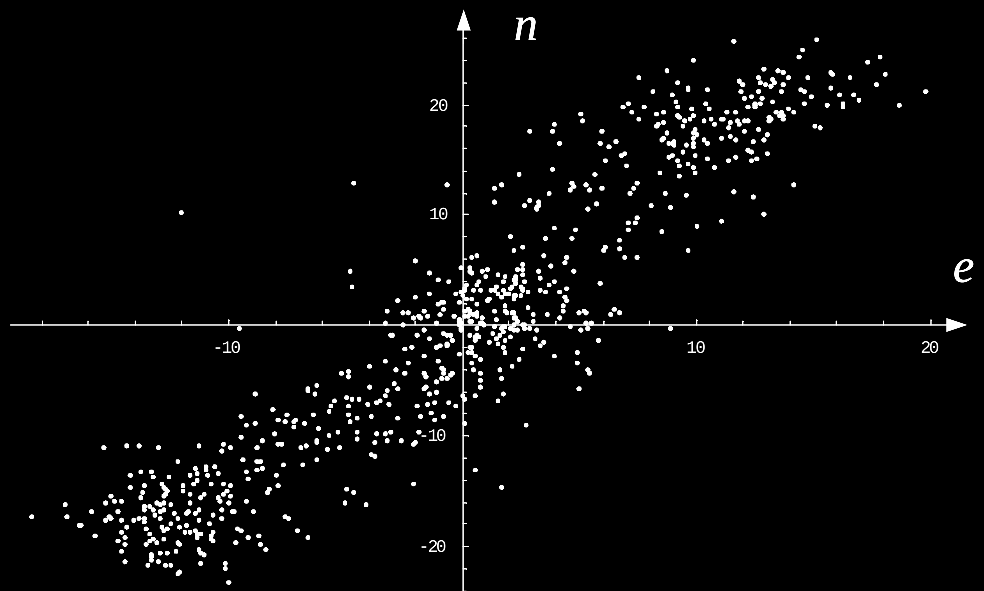
GPS observation time series

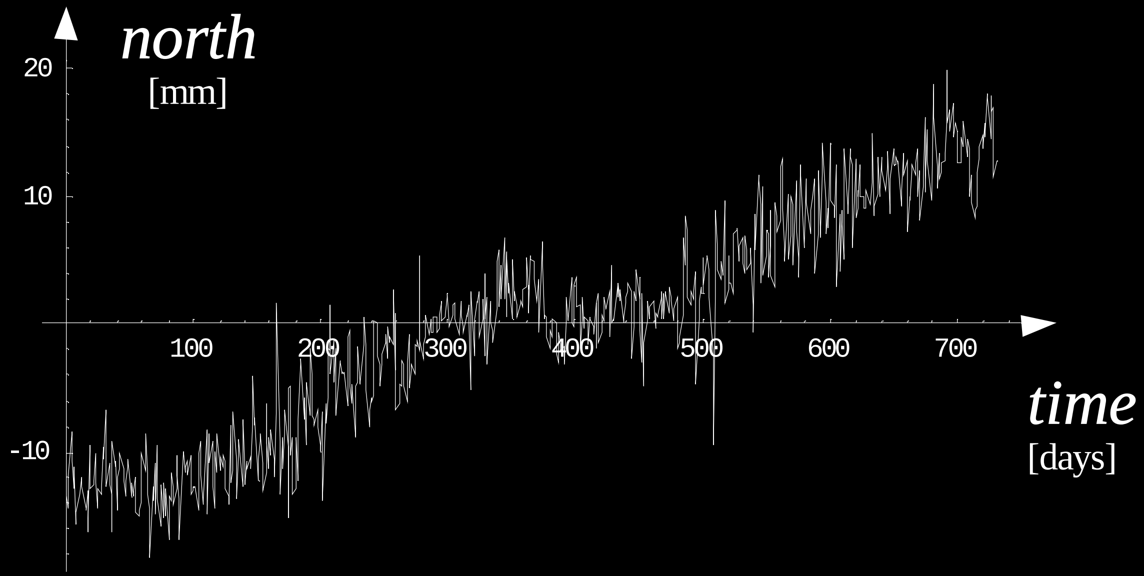
Location: Permanent station Borowiec (Poland)
Part of: EUREF Permanent Network
Purpose: Regular monitoring of recent kinematics of the Earth's crust (among others)
Period: 2001-2002
Time points: 730 daily average values
Coordinate system: Horizontal ***(n e v)***

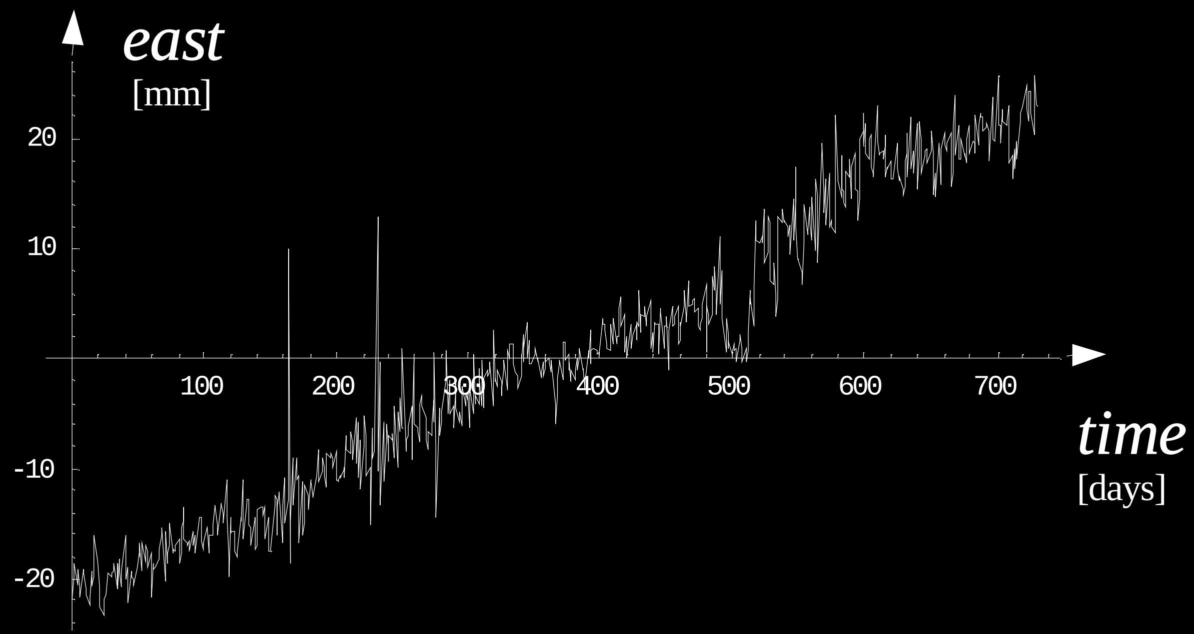
3D - scheme:



2D – horizontal plane:







Note:

The obvious linear trend is the consequence of a long-term drift of the Eurasian tectonic plate.

Processing

3 ways to treat our data:

- Process it separately as two independent univariate time series.
- Accept the interrelationship and use the multivariate modeling methods.
- Test for cointegration and transform the data with respect to common trend.

The goal of our experiment:

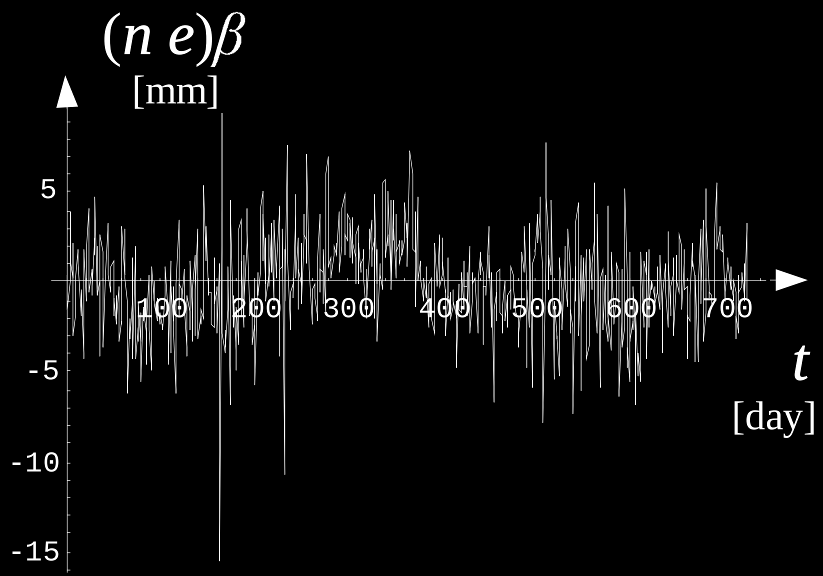
Compare the three approaches according to forecast accuracy and show the benefit of the cointegration theory.

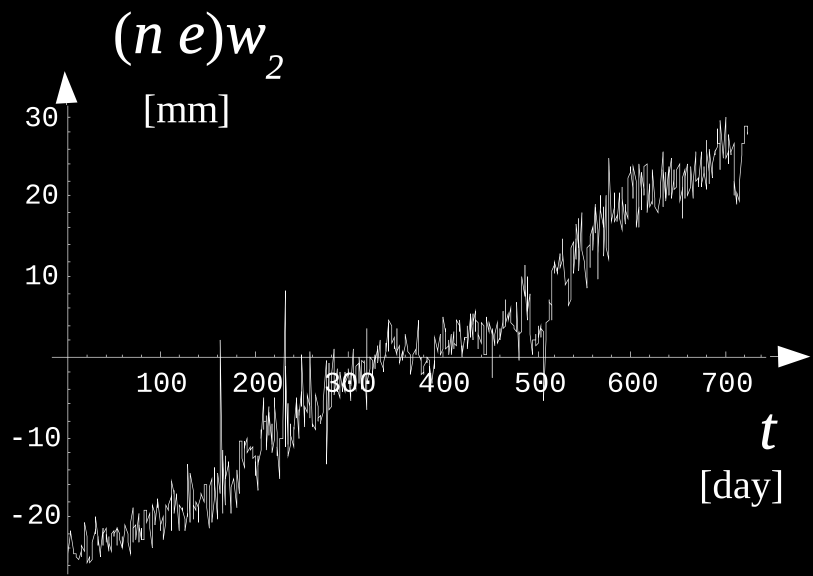
Cointegration

Two non-stationary I(1) time series are cointegrated, if one of their linear combinations is I(0) and hence stationary. I(k) denotes “integrated of k-th order”.

Procedure:

- Testing for the presence of stochastic trend (against pure deterministic components)
  - tests for unit roots (Dickey-Fuller)
  - stationarity tests (Kwiatkowski)
- Testing for cointegration
  - Engle-Granger two steps method
  - Johansen’s method
- Common stochastic trend estimate (Gonzalo-Granger)





Equilibrium (or long-run) relation between n, e.

Common stochastic trend.

Note:

b denotes cointegration parameters vector and w_2 eigenvector pertaining to common trend direction.

Geometric viewpoint

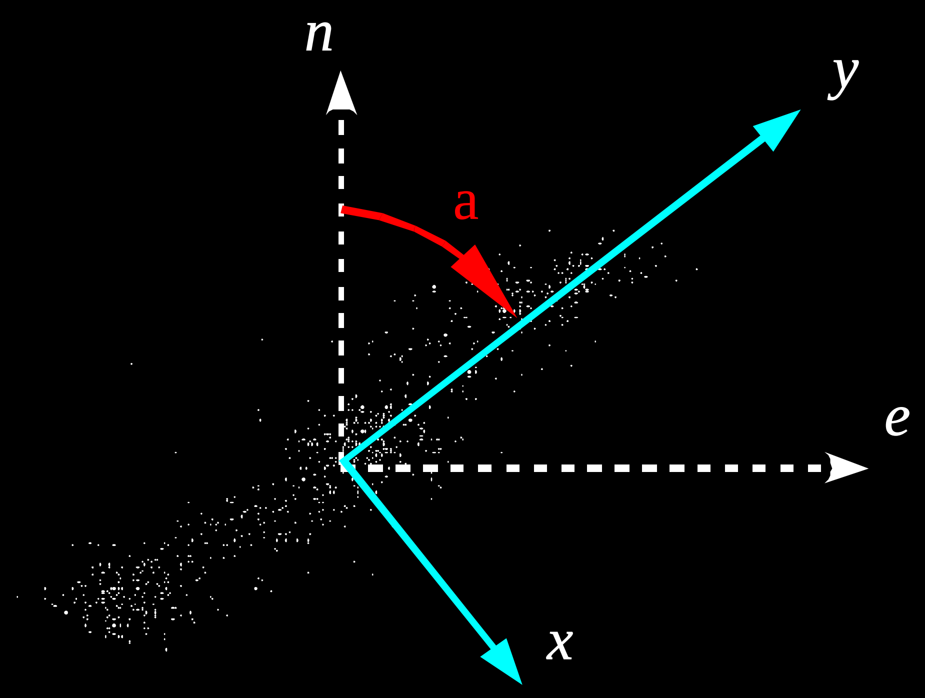
Looking for linear combination

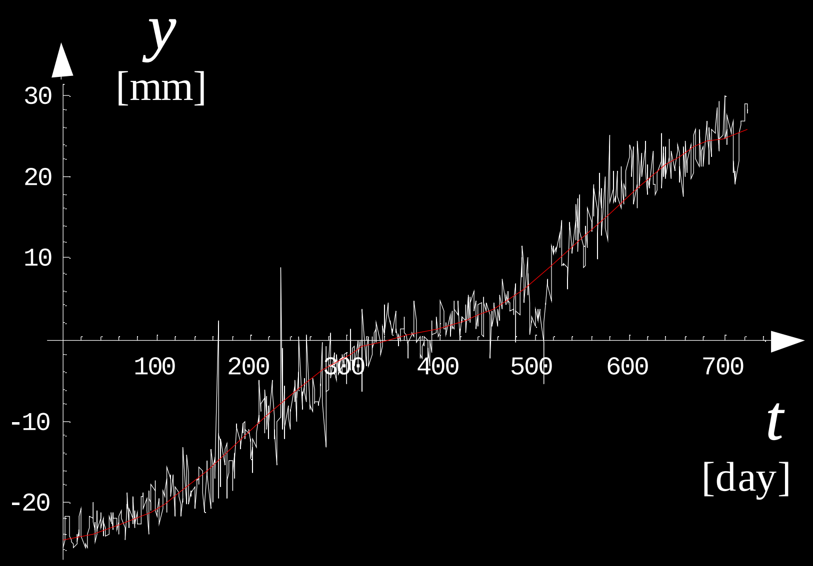
$$y = g_1 n + d_1 e$$
$$x = g_2 n + d_2 e$$

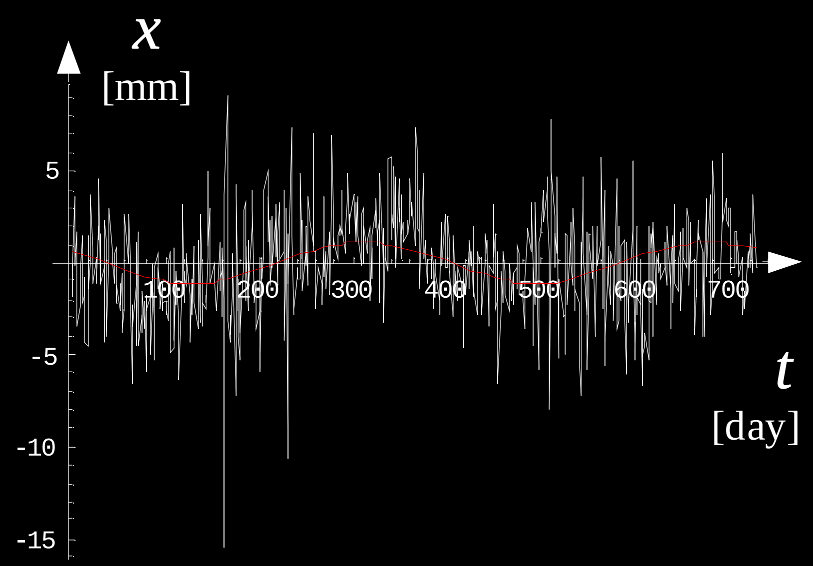
such that y represents a common trend direction and x a stationary trend-free variable, orthogonal to y, the problem is easily rewritable into familiar transformation

$$\begin{bmatrix} y \\ x \end{bmatrix} = \begin{bmatrix} \cos a & \sin a \\ -\sin a & \cos a \end{bmatrix} \begin{bmatrix} n \\ e \end{bmatrix}$$

The angle a can be determined from analysis of deterministic (or stochastic) trend.







Common trend direction variable containing linear trend and seasonality.

Trend-free variable contains only seasonal component.

Final steps:

- Apply Box-Jenkins methodology.
- Transform the whole model back to north-east system.
- Forecast 5 out-of-sample values.
- Compute mean square (MSE) and mean percentage (MPE) error from model prediction and known real values.
- Compare errors from all of the methods.

Results

variable	trend [mm/year]	seasonality	
		amplitude [mm]	period [days]
n *e*	13.2 21.5	2.2 1.6	365
y *x*	25.2 0	2.5 1.1	

Deterministic model parameters

method	variable	order p	mse [mm^2]	mpe [%]
1.) independent univariate time series	*n*	1	7.40	5.08
	e	4	3.70	2.88
2.) multivariate time series	*n*	2	8.13	5.44
	e	2	5.06	5.13
3.) respecting common trend	*n*	2 (*y*) 4 (*x*)	**5.90**	**4.04**
	e		**4.10**	**2.49**

Forecast measures of model effectivity

Thank you

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